Good afternoon, today we talk about lecture 10, network.
As I mentioned last time, right now it's a good timing for you to review linear system
and Fourier analysis.
We are going to have first examination on February 23rd.
And I discussed with the other instructor, and we basically cover the same stuff, but
we have the book draft to follow and a little different, and we emphasize network also in
different way.
And in this lecture, I will cover not only electrical network, and I will also explain
a little bit about artificial neural network.
So for this blue part, mean the foundational part, this part, we will do examination using
different questions, but after first examination, and in the last two examinations, we are going
to solve pretty much the questions.
So this is the general concept.
And also this morning I uploaded, revised, not thoroughly revised, mainly focused on
the signal processing, this created Fourier transform.
So there are some typos I fixed, supposed to be clear.
And also when we explain sampling theorem, and I used some deeper concept in the main
text, but now I move that part to the remark section.
So anything in the remark section, you can read lightly, so get additional knowledge.
But the basic formulas and the scales should be well mastered.
So that's just something you can expect in the examination.
And typically, I like to give you multiple choice questions.
So you can just do some not so complicated computation, or use the knowledge you learned,
the conceptual understanding.
So you can make sure, show me you really know this part, the convolution, transform, and
the sampling signal processing, and so on.
So the network kind of built upon what we learned.
In the next lecture, we talk about image quality, and how you judge an image is in good quality,
and how you relate your image quality to diagnostic performance.
So the next lecture, I mean this one, will be very useful to all the imaging modality
I'm going to teach.
So just the network and the quality lectures, then we really into the imaging modality content.
Talk about X-ray, CT, and just five major modalities for this lecture.
So now we talk about the network.
And before the network, we explained the system, linear system, even safety environment linear
systems.
And I mentioned function, more mathematics-related stuff.
And the system and the kind of engineering-oriented.
Function and system, they are kind of the same thing.
And just a different terminology.
You have input, then you have output.
And for function, you can talk about range and the domain.
For system, often we talk about the input and the output.
Why we have the network, and what's the difference between network and the system or functions.
And I would say system can be simple or complicated.
The network usually means a more complicated system.
And you have many subsystems and interconnect them together.
So you make a network.

Like internet, you have a computer and many, many computers linked together, communicate
all precise information.
So you have a network as a very popular concept, complicated, interconnected subsystems.
And if you view from a mathematical perspective, a function is also a multivariable relationship.
So that's just a complicated mathematical function.
And the function is not as simple as y equal to f of x.
So the function can involve a million, billion variables.
So this network can be, indeed, represented as a multivariable, complicated function.
Oftentimes, composite functions are involved.
So you have some input fitting into another function.
And all these functions are connected together.
So these three concepts, I would say, they are really the same thing in a sense.
And the network can be very simple.
That's just a simple network.
But usually, networks are quite complicated.
And then normally, when we learn network in electrical engineering,
we talk about electrical network.
So you make a network use electrical components.
Typically, you know, register.
And later, I will show you capacitor, inductor.
You link them together.
That's an electrical network.
And nowadays, artificial intelligence.
And then they talk about artificial neurons.
And then you link many, many neurons together.
Then you also have a very complicated network.
Electrical network and neurological network, they are also closely related.
And the neurological system really works as an electronic system in a big way.
Because the signals in your neurological system
really transmitted as so-called electrical potential.
So just the electrical pulses send down the neurological fibers.
And then we will see that in minutes.
But anyway, so let's start with more classic engineering stuff.
Electrical network.
So basic components are three.
Register, capacitor, inductor.
For register, you have ohms load.
And you already know this.
So V, the voltage across the register, is a product of the current passing through under the resistance of these components.
In other way, you can say, OK, I equal to V divided by R.
So I, the current, is proportional to the voltage applied across the components.
It's inversely proportional to resistance.
That's why you call it resistance.
So it just resists the voltage.
So they try to restrict how much current or how many electrons you
can send through the resistor.
And a more advanced component, capacitor.
So something shown here, this is a capacitor.
You have a voltage source.
This is a battery applied across the capacitor.
The result is that the electrons will be driven towards one plate
of the capacitor.
So the other plate will also form a current going this way.
But really, the capacitor just blocks electrons, really goes through.
But driven by the electrical potential,
the result is that positive and negative charges
will be accumulated across this capacitor.
So the positive charge here, negative charge here.
When the amount of the pure positive or negative charges

accumulated enough, so it will counter-react
to the electrical potential.
Then you form a dynamic balance.
So mathematically, we say the current is equal to capacitance
C times the first derivative of voltage with respect to time.
The other way, just put it in the integral format,
something like this.
So basically, you just imagine electrons
are driven towards the capacitor.
Or just draw away from the capacitor.
Depends on which plate you are talking about.
So this is just the relationship between voltage and current.
They are not just straightforward.
Just the I equal to R, V. It's not like that.
But they're really linearly proportional,
directly proportional to the first derivative.
So this is for capacitor.
On the other hand, for inductor, it's something very similar.
So you have the direct proportional relationship
between V and the first derivative of the current.
Here, you have a first derivative of voltage.
You just exchange the positions of voltage and the current.
So on one hand, you have a capacitor.
On the other hand, you have inductor.
So you have an integral relationship.
You have a differential relationship.
So these three are very important components
from classic circuit.
And to understand these relationships better,
we need to review so-called phasor.
And just revisit complex numbers, things like that.
So for complex numbers, we have a real part, x.
And we can have an imaginary part of y.
So you can represent an arbitrary point
on the complex plane as a complex number.
So real part and the imaginary part, x and the y,
something like a Cartesian coordinate system.
And also, you can use the polar coordinate system.
So instead of representing this particular number,
complex number, as x, y, you can say about the radius r
and the phase angle phi.
So this is an alternative representation.
And you can just denote it as r times e to the power i phi.
And e to the power i phi is a complex exponential function.
And we learned Euler formula.
And this part, e to the power i phi
equal to cosine phi plus i sine phi.
So if you just multiply r with the sine and the cosine,
and you just return to this part.
So they are really equivalent.
And this is something show we have
an essential quantities, radius and the phi.
So if we talk about the sinusoidal vibration
that we learned in Fourier analysis,
and we know the essential quantity is r and the phi.
So we need both quantities.
So why not just put these two numbers simply?
Do not bother write e.
Do not need i.
Just say r, the angle, the phase angle is phi.
And this is just, I think, the simplest notation.
And then we call it a phasor.
So any sinusoidal vibration, you can always just use the phasor,
assuming you already know the frequency.

Like the AC current, you know the frequency, 60 hertz.
Then the rest will be specified by r.
That shows how strong the current is.
And also the initial angle phi.
So that's the initial phase angle.
Just show when you start, given a coordinate system.
In this case, so the sinusoidal wave
started a little bit earlier.
So you have this phase angle.
Phi divided by angular frequency omega.
Angular frequency omega equal to 2 pi f.
f is a frequency.
So these are just the basic quantities.
And we will see this is a very convenient notation.
And now it's an opportunity to just think a little more
about a complex number.
And I once mentioned that in complex domain,
and you really utilized this imaginary unit, i.
So you may feel this i, very first place,
defined as a square root of negative 1.
Any question?
So square root of negative 1, it looks not very reasonable.
So really, this square root of negative 1
plays an important role.
So let's just revisit it a little bit.
According to Euler formula, you have this one.
e to the power i theta.
You can decompose into real part and the imaginary part.
So if you just make an angle theta, 90 degrees,
then you see this i is just this part.
e to the power i, 90 degrees.
So if you have a unit vector, this one here,
then you multiply this unit vector with this i.
The result is that you turn this by 90 degrees.
So you make this unit vector 1 becomes i.
You see, if you have 1 times this.
And if you do this again, so you just
do the 90 degree rotation twice.
So this can be decomposed into e to the power i,
90 degrees times e to the power i, 90 degrees.
And I argue that this is i.
So you've got i times i equals minus 1.
So in this way, you can understand
that i is a rotator from a positive 1.
You rotate twice, then you get negative 1.
So then we say you do this twice squared.
Then naturally, you think i is really meaningful.
This is the square root of minus 1.
So this is a geometrical way to understand this mystery i.
So if you have a unit rotator, e to the power i theta,
that means you rotate this unit vector by angle theta.
So this angle is theta.
Then you do multiplication with times i.
So just put times r.
So you've got r here.
Then you have r times cosine theta, r times i sine theta.
So this shows that multiplication means you're just scaling it.
If you multiply this r, you scale it by this way.
If r is a real number.
But if you just multiply this e to the power,
this phase factor, that just means rotation.
Suppose you have two complex numbers
on the complex domain.
You have u, you have v. Then you add them together.

So you add the real part together and the imaginary part
together.
The result is just this overall vector, vector u plus vector v.
So from this picture, you see how you
get imaginary unit i.
And with that, you can define very reasonably addition
and multiplication.
So if you can define addition and multiplication,
then you naturally have the definition
of subtraction and division.
So you have a whole algebraic system for complex numbers.
So this way, it makes life easier.
So we know this imaginary number or complex number
are really reasonable.
And the phase representation, particularly good
for sinusoidal waves.
And the sinusoidal waves play a fundamental role
for Fourier analysis.
And the wave equation, later we will
learn for ultrasound wave for electromagnetic field.
So the sinusoidal wave is really a fundamental building block.
So these things come together.
And we see it is so important to have Fourier analysis
and the phase representation.
And the phase representation is applied here immediately.
You see how we can extend Ohm's law.
Ohm's law is nothing but a voltage and a current
relationship, or VI relationship.
But for capacitor, we show the relationship
is in terms of difference or an integral operator.
But actually, with phase representation,
and we can have the relationship as just the same thing,
the multiplication, like Ohm's law you learned in high school.
And here we have some important claim here.
If we use resistor, capacitor, and the inductor
to form a circuit, any circuit you form,
so then you drive the circuit with the sinusoidal
excitation, sinusoidal signal, current or voltage.
Then the whole system will only have a sinusoidal wave
form at the same frequency.
Remember I explained to you about convolution theorem.
And I have one slight argument.
For linear safety environment system,
and your input is sinusoidal wave at a certain frequency,
the output must be at the same frequency.
The amplitude and the phase here,
amplitude and the phase can be changed,
but the frequency will not be changed.
So we argue that you still remember.
That's very good.
So in this case, we say if you have a capacitor,
you drive the components with a sinusoidal voltage,
then the current must be sinusoidal at the same
frequency.
So suppose the voltage is sinusoidal.
We call it A is amplitude times cosine 2 pi FT.
Frequency T is time, and the 2 pi F is frequency.
Then omega is angular frequency, because you have 2 pi here.
So the voltage is A times cosine omega T.
So this is the voltage.
Then what is the current?
The current, I told you, you have a differential
relationship, I equal to C times the first order derivative.
So you perform first order derivative,

you have this part.
You have sine omega T. And sine omega T
can be converted to cosine omega T plus phase angle.
In this case, the phase angle is 90 degrees here.
Looking at this, so we say this is a sinusoidal form.
And with the amplitude C capacitance times omega times
A, this is the original amplitude.
Then with the phasor notation, the angular part is 90 degrees,
because this is pi over 2.
So you've got this phasor representation.
The phasor representation corresponds
to a complex number.
So if you change to complex number,
you will see this blue notation in phasor
is equivalent to this right relationship
in complex number.
So the V equal to something times I, this something
is really plays the same role as resistance.
So this is called a capacitance.
So you have one over J. J is an imaginary unit.
And you could put I here.
But in electrical engineering, people usually
use J instead of I. Then you have this omega.
The omega depends on frequency.
But I already told you, we assume the frequency is fixed.
With single given frequency, the voltage and the current
will share the same frequency.
And another factor is C. So this is the VI relationship.
If you look at the right formula,
this is not a differential relationship anymore.
And just in the domain of phasor representation,
so V is equal to a complex constant times I.
So just the same way, like V equal to R
is a real number times I. But here V
equal to a complex constant times I.
And very similarly, so for inductor,
so this is current sinusoidal form.
What's a voltage?
Voltage is equal to L times the first derivative of voltage.
So you just perform the differentiation.
You got this part.
Then just do triangular change.
Then you see, OK, this current in phasor notation,
B and the angle 0.
Then after you do all these steps,
so V is equal to a changed amplitude
and a changed phase angle.
Again, this is a phasor notation.
So in phasor notation domain, you know the initial I.
And given this initial I, you can compute what will be V.
So V has changed the amplitude and has a additional phase
angle here.
Again, the phasor notation is equivalent to real complex
notation, which show in right.
So V equal to j omega li.
So this j omega li put together is a complex constant
in the position of R for Ohm's law.
So you generalize the Ohm's law.
So we say, instead of just call any effect
of the electrical component resistant,
we just don't call resistance.
Resistance only related to resistor.
Now we have three kinds of components.
Resistor, capacitor, inductor.

So how about we gave a broader concept called impedance.
So impedance is more general than resistance.
So it covers resistance, capacitance, and inductance.
So each of them, in terms of VI relationship, V divided by I,
if you only use resistor, then this is Z. It's just R.
But if you link three kinds of components together,
then you can have the resistance part, capacitance part,
and the inductance part.
So you have these three colors.
And you can link them.
If you just use them individually,
then we derive the VI relationship.
And you can combine them together.
And it depends on the type of connection.
In parallel or in series, you can compute equivalent impedance.
And we will come to that later.
Before I explain how could we compute equivalent impedance
in series or in parallel network.
And let's just say a general electrical network.
Just to give you some graphical concept.
For general network, in particular,
electrical network is just a simple example.
So we can define three important concepts.
Node, branch, and loop.
What is a node?
Node is a point where circuit components are connected.
Circuit components, again, can be battery,
can be resistor, capacitor, inductor.
And the branch is a circuit component between two nodes.
So just make a path called branch,
or just an arm reach out from one node to the other node.
And the loop is a closed path through any node, branch,
only once.
You cannot go from one end of a branch to the other end.
Then come back.
That's kind of redundant.
So you just need to bypass nodes and branch only once.
So in this circuit, you may ask, how many nodes?
According to the definition.
And how many loops?
How many nodes?
We see this is a node.
That's a fixed definition.
Because this is a point connect this one branch,
this other branch.
This is a node.
Because these lines are just a perfect conductor
with zero resistance.
So we don't care.
So this overall, this is a node.
So you'll use this node to connect R1, R2, R3.
And also, this voltage source.
And here is another node.
So just to show you the concept of the node.
How many loops?
And really, I mean, how many independent loops?
So we say independent loops are three.
This is one loop.
So just transverse each component
in this small loop once.
So battery, the resistor 1, resistor 2.
So you do this once.
Then this is the second one.
And you could argue, this is also a loop.

Yes, this is a loop.
But given you already specify this is a loop, this is a loop,
then this loop can be derived from the previous two loops.
Then you have a third loop.
So these three loops, if you go around each of them,
each time, you have something new.
That's why I call it independent.
If you just use these three loops,
then you do this treatment, then nothing new.
Everything you already have seen before.
So why we need this concept?
Because when we solve a complicated circuit,
there are multiple unknowns involved.
For example, what's the voltage across this resistor 2?
And what's the current going this way, going this way?
So all these are called unknowns.
To solve these unknowns, we need to list
a system of linear equations.
And to have these equations, equation
means something equal to something else.
Then you need some consideration.
First such consideration is called
Kirchhoff's current law, KCL.
This is very, very straightforward.
Just say for any node, I just explain to node.
But here is a node.
Then you think the sum of current into this node
must be the same as the sum of the current out of the same node.
So therefore, this node is just an abstraction.
It's something there.
And you think the situation, the current really electrons
moving around.
Some electrons are moving towards the node.
It will not stay there.
They move at high speed.
Then they go away.
So if you look at any node, then you
can claim the net sum must be 0.
This is something like we have a room,
and then you have some student come in,
some student come out.
So in a dynamic picture, so this ought to be balanced.
The number of cars interlay some circle with nodes
must equal to the number of cars moving out.
You cannot park your car there.
So this is just something like a conservation of matter.
So just one way.
The second one is called Kirchhoff's voltage loop.
Basically, it says if you go around a loop,
then the voltage drops.
And the driving power, just the battery power or voltage
shots, they sum together.
Then the total must be 0.
This is really like an earlier picture.
And we just use an analogy.
The current flow, like just a water flow,
you go from one place, just flowing around, then come back.
Like you just walk, climb a mountain.
So just you go from one location.
So this is you just do a tour.
So from one location, you're just climbing here.
So climbing up, and then you walk around,
then just come down, return to your original position.
And then you high school physics,

you know you move around, the total energy,
the potential energy will be 0.
You move up, you gain higher potential energy.
You move down, you lose energy.
So the water flow or the physical potential energy
conservation applies here.
And this is not a gravitational field.
This is really electrical field.
And then you have electrical force,
driven electron moving away.
That's the same thing.
So you'll gain the potential between two points
in a non-trivial electrical field.
But if you move around, go all the way around the circle,
the total energy, the potential energy,
will just increase, decrease, and finally
will all become canceled out.
So the potential difference around the closed loop
must be 0.
So this reflects the consideration
of energy conservation.
So in this case, you see, you have this driving force,
voltage source, V1 equal to 10 watts.
Then this voltage drops across this register.
So you have a voltage drop, 2V, voltage drop, 3V.
And here, it's just a pure conducting wire,
no voltage drop, because resistance is 0.
And here, you've got a third drop is 1.
That question is, what is the voltage drop
across the third register?
It must be 4, so that you can have algebraic sum 0.
So this is another consideration.
So with these two KCL, KVL, with these two relationships,
you can list enough number of equations
to solve the complicated electrical network.
So think this is true.
If you just have one loop, you can solve anyway.
This is like V equal to Z or R times I.
So you solve immediately.
Then you can think from your very simple network.
You keep adding more loops or nodes or both.
Each time you add more loops, each time
you add one loop or one node, you have a new loop,
and you can list a new equation.
You add one node, you can add a new equation.
And then you can add both loops and nodes simultaneously.
Then you can have more equations.
Let me show you what I mean.
So I say KCL plus KVL can guarantee
you have a unique solution.
We have a starting point.
I already explained that just with a single loop,
a simple configuration, you can solve uniquely.
But how about let me just add one more loop.
And then we say the original network already
allows you to do a unique solution.
Now you add this one more.
So the number of unknown is increased by one.
But you add one more loop around the loop.
You can use KVL.
So you get additional equations.
So you can still have enough number
of independent equations to solve it.
Then let me show you a case.

You can add a node without adding a loop.
So look at here.
Here you have this branch here.
Let me move this branch.
So just pay attention to this part and see what happens.
So I move this part to here.
So you add one more node.
Then you have the same number of loops.
Because the loop number is the same.
It's just an additional node.
Just see here.
You have just a loop locally.
You have one loop, two loops.
But I change it this way.
So still one loop, two loops.
But you get additional nodes.
And you need to solve a new unknown.
So number of unknown is increased by one.
But you have one new node.
And for this node, you can use KCL.
You got a new independent equation.
You still have enough number of equations to solve the system.
And this case, this blue case, you add one more node.
At the same time, you add one more loop.
So you have two equations.
In this case, you really introduce two unknowns.
Why is this?
So originally, you have only one unknown here.
But by introducing this new node,
so this becomes two unknowns.
So you have this part.
Originally, the whole branch, you
have a current going through.
But you introduce this node, the chance
is that the current going this way and this way
is no longer the same.
So you introduce two unknowns.
But you have one new node, one new loop.
So you've got two more independent equations.
You still have enough linear equations
to solve the whole system.
So this argument can be built on from just a simple case.
Just keep adding.
It just shows you how you consider the problem.
So you can feel free to use KVL, KCL as needed.
But still, you have enough number of equations
to solve it uniquely.
So this is an example.
How you solve the electrical network like this
is already more complicated than what high school students show.
So you have two batteries.
So they just counter-react.
You have two loops.
So if you ask a high school student,
they may not be able to solve.
They only know single loop.
It just uses the Ohm's law.
But here, we have two elegant considerations
called the KCL and the KVL.
So two things we can use as we see fit.
And we can do it systematically.
But for our purpose, just based on our intuition,
see which way looks easy.
So just keep adding independent equations

until the number of unknowns and the number of equations
are the same.
Then we can solve the problem.
So in this case, we target three unknowns.
It's I1 going this way.
I2 go this way.
I3 go this way.
And looking at nodes, this right node, we say I1 plus I2.
So these two are currents into the nodes.
Must be equal to I3.
That's just out of flux.
So you've got the first equation.
You need two more.
You see, you have two independent loops.
You can either consider this way, this one loop,
this second loop, or you consider this is a loop,
this is the second loop.
Why I choose this small one?
Because the smaller loop just feels easier,
just the arbitrary feeling.
So for the green one, you see, you need to pay attention.
You have freedom to specify the direction of the loop.
And we just consistently make counterclockwise,
not counterclockwise.
So if you say this is clockwise, then this battery, 14 volts,
is against your direction, the loop direction.
So you have a minus sign here.
And this second battery, again, is against this green direction.
You've got a negative sign here.
Then the current I1 here, so you have a voltage drop here.
Voltage drop against this direction,
you have positive.
The other one is 4 volts.
So the current I go this way is in the same direction
of this green direction.
It's a clockwise direction.
So you have a minus sign.
So this is a good example, very simple.
And after class, you just check.
You make sure plus, minus, so don't get confused.
The essential idea, really, you give a direction.
Then you ask the voltage drop sum together.
And the driving voltage source added together,
they must be equal.
So you can say, in this case, the voltage source direction
is against the loop direction.
You say it's negative.
Then you consistently do so.
You could, in the very first place, you say it's positive.
Then the voltage drop, you need to take the other side.
But after all, the end result is the same.
You just make sure the two opposite effects added
together, and they will be 0.
And you apply same consideration to this blue loop.
You've got this equation.
So you can just review it yourself.
And this is a simple thing, but you try to make sure.
Pay attention to direction.
And you apply KGL and KVL so that you
have consistent results.
And my suggestion, you may just copy this part.
Do not view the right-hand side.
You try to solve.
You list the equation.

See if you got the same thing as what I show here.
And if you don't, then you just dig
into the directional differences.
So this kind of summary is almost all generalized.
So originally, you only have V equal to R times I.
Now thanks to phasor notation, we
know V is still equal to some constant times I.
This is some constant, and depending
on which components you are talking about.
If you just use a register, you return to almost all.
If you use a capacitor or inductor,
and you have just Z equal to different thing,
different for capacitor, you have this one.
For inductor, you have this one.
So in phasor notation, so you have
leading 90 degree, and then you have lagged 90 degree.
So just think about this.
It's a little confusing first place.
But if you are electrical engineer,
you know for inductor, you cannot pass current immediately.
So always the voltage applied immediately.
So the voltage first, then current graduated building.
For inductor, you cannot have voltage
applied across the inductor.
The current is big.
But the current will be less and less
as you accumulate more and more electrons on the plate.
So this is just engineering intuition.
But mathematically, you do the derivation
to got this part.
OK.
Now, how do you do the computation
of equivalent impedance?
Along a branch, you have two impedance values, Z1 and Z2.
And then we can find the equivalent one.
We say just like a resistive circuit,
you have R1, R2, the equivalent resistance is R1 plus R2.
So this Z equivalent is equal to G1 plus G2.
And in parallel case, you got this reciprocal relationship.
So reciprocal of equivalent impedance
is equal to sum of 1 over G1 and 1 over G2.
This is exactly the same thing.
Because the voltage law is, because the generalized Ohm's
law is the same.
And how can you prove it?
It's very easy.
So let me just verbally say, this may be your second,
not homework, but this is second thing.
You want to work out yourself when you review these slides.
You think you are passing a current I through this branch.
And the same I going through this.
Then we say the voltage, this part and this part
are viewed as a black box.
So you have the current going through.
You have voltage V. The same voltage
V cross this part.
Then you know for this one, just look at this.
Then the V must be equal to I times this Z equivalent.
So this is on the right-hand side.
On the other-hand side, you say this V
must be equal to voltage drop.
The current going through is the same.
Must be equal to voltage drop.
Here is I times G1, I times G2.

So V is the same.
Because these two blocks are supposed to mean the same thing.
V is the same.
Then you express V in a different way.
You just let the two parts equal.
Then you factorize I, you get this one.
So this is something you can prove.
And here, the current is not the same.
This is I1, this is I2.
But the voltage is the same.
Here is voltage is the same current.
In this case, the current is the same going through.
The voltage is equal to sum of voltage drop
across Z1, across Z2.
Here, overall, the voltage is the same.
The total current is equal to I1 through Z1,
and I2 through Z2.
Then you just do this transformation.
Show the equivalency.
You will derive this.
Just the one step.
We have 10 minutes rest.
Then we finish the next part.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
Thank you.
So with equivalent impedance, you
can imagine you have a network for any branch,
multiple components.
You can always combine them into a single component.
So each branch, you can have an equivalent impedance only.
Or you only have electrical power source, either voltage
source or current source, or both.
So at most, two components.
So if anything more, then you can try to combine.
So once you have the equivalent concept,
you can simplify that work.
And the current flow inside the parallel connection,
and then you can compute it easily, as I explained,
as long as you know the overall voltage.
Or the voltage drops through a series branch,
you can compute it easily.
So just do the combination.
So you'll have an equivalent impedance first to solve
for the network.
Then you can distribute the results into each component.
So this is just a two-step thing.
And if you just don't care, you just

solve the overall network.
That's also fine.
So with the impedance as a general concept,
then we can do the voltage divider.
Like this, you have an input voltage.
The voltage will be divided between R and C,
this capacitor, according to what we learned previously.
So you can compute that the transfer function between input
and output is just shown here.
So this is the overall voltage drop
will be divided between R and C.
So the input voltage times this transfer function
will equal to the output voltage.
And you see this transfer function
serves as a scaling factor.
And this scaling factor is a frequency dependent.
The omega C is 1 over RC.
This is known as long as we know R and C.
And the frequency, the sinusoidal wave
you inject into this circuit can be changed.
The frequency can be 60 hertz, it can be 100 hertz,
and so on.
Here is a good example of Fourier analysis.
So you have an arbitrary waveform here.
Then you perform Fourier analysis.
So you decompose your input signal
as a summation of many sinusoidal waves.
So it depends on the frequency for a given wave.
You can compute output voltage at that frequency.
And you have many, many frequency components.
And using this formula just for corresponding frequency
or angular frequency, you will have a corresponding output.
So the input is a summation of many waves.
The output is a summation of many waves.
So you just do the computation, frequency by frequency.
So this is just the electrical counterpart of Fourier analysis.
So you really can compute this way.
And you see the response depends on frequency.
And for certain circuits, and it looks like this one,
we call it a low-pass filter.
Why we call it a low-pass filter?
Because the frequency components,
when they are low, zero or very low frequency,
slow changing things will be transferred better.
If you have very high frequency, see this omega is very high,
then this scaling factor, the transfer function,
will be very small.
That means you have input, say, 100 volts times t,
because omega is very big.
So t is very small.
So output will be very small.
So as shown here, you have an input signal.
You do Fourier domain decomposition.
So you have a certain spectra in the Fourier domain.
So lower portion, you basically let it go through.
Higher frequency components, you block it.
Why you block it?
Frequency is high.
This number is small.
So this is called a low-pass filter.
Low-pass filter does not look like a rectangular function.
It's rather something like a smooth Gaussian-shaped function,
something like that.
It's not a Gaussian either.

So this transition point is denoted by this omega c.
See here?
This omega c.
So omega c is known.
So omega c kind of like a bandpass,
how wide you can let the signal frequency transmitted
to the next stage.
So at omega c, the input power at that frequency
is dropped half.
So they just say, like later on, we
mentioned the resolution measurement.
So we say this is a place.
And then we think the signal dropped low enough.
And then we think only within this range,
roughly, signal can go through.
And beyond this range, you block the signal, not absolutely,
but just the proxymission.
So this is a low-pass concept.
So low-pass, on the other hand, you have a high-pass filter.
And you let a high frequency go.
Or you even have some bandpass, only frequency
within this range.
You can let it go through the circuit.
Or you have a band stop, so just a block of frequency
component within a certain range,
just all kinds of applications of Fourier analysis.
So you decompose input into some of sinusoidal components.
Then you use the impedance concept,
compute system response at each frequency.
They'll add up together.
So originally, we say we just use the circuit analysis
to compute a given frequency response.
But now we say, and we do use Fourier analysis.
We can just compute the input and the output
for arbitrary waveform.
So this is just a way we solve a complex circuit problem.
We deal with a single frequency first.
Then we build our result on that solution.
So this is something very, very important.
You can view it as an application of Fourier analysis.
So for direct current analysis, circuit analysis,
DC circuit analysis, we use almost all.
So everything is just a linear equation.
Something times just the current times resistance.
You have a voltage, so all purely linear equation.
But this is sinusoidal input or sinusoidal power source.
You really need the differential integral relationship.
And the solving differential equation
is more complicated than solving algebraic equation.
But with the phasor notation, the sinusoidal response
can be simplified in the phasor notation.
So it looks like algebraic system.
Just the v equal to z times i.
So we can solve algebraic equation
instead of differential or integral equation.
So in that sense, we say the AC alternating current circuit
is just the same thing as a direct current circuit.
So AC is DC in this case.
So this greatly simplifies circuit analysis.
And this idea was invented by an engineer
in GE Global Research Center several decades ago.
Now far from here, GE Global Research Center.
So this is a very important computational scale.
So next, let's just show you a very typical circuit.

Again, this is something about voltage divider.
So you have an input signal.
So you just link to the next level.
So part of the voltage is divided between two impedance
or two resistors.
And you have a voltage drop across this part.
And this is linked to the next level.
And the circuit like this is important.
So the application is you have a signal here.
You want to detect.
Then you do something related to the next level,
like signal processing.
This is a sensor.
So this current, this resistance or impedance
is necessary.
Like you measure cardiac signal, the human body
would have an impedance and a resistance.
So you have a signal inside your heart.
So go out.
So you have to have this.
You line up using the equivalent concept.
You find the equivalent impedance.
So you represent that as R1 or G1.
So you have this part.
Then you want to take a signal out.
You have to let the voltage drop on something.
If it is zero, you don't have signal.
So you need to have an interface with the next instrument
or circuit.
So you need to have this one.
Then you put it to next stage for further processing.
Signal processing system or network
can be decomposed into several stages.
So this is just a very much simplified picture.
So we have three very, very practical, reasonable
considerations.
First, we hope the input resistance is large.
So we want this resistance is large.
Because only when it's large, it will for the same current
going through.
So this impedance or resistance large enough
so you can attract a larger portion of the signal.
So from this part, you have signal.
You want to make this very big.
So you basically have 100 or near 100% of signal drop,
voltage drop, across this component.
This is one consideration.
But once you have this signal, you
want to relate to the next instrument, the next level.
In this case, I just want to have a very small resistance
or impedance here.
Why I want it small?
Because the equipment or interface or circuit here
want to take a signal out.
So if you have a small resistance,
and we can get a bigger signal.
If you have a very big resistance,
and you cannot get a whole lot of things
because the resistance here is just too big.
And also, if possible, the signal usually small.
And if I could magnify the signal,
make the small signal very big, that will be very desirable.
So these are practical considerations.
And we got kind of self-contradicting.

We want from one consideration, we want this big.
The other one, we want this small.
Anyway, we want to magnify useful signal.
These things need engineering balance optimization.
So we need to find good ways to design circuit
so the conflicting requirements can be altogether optimized.
For example, this is a circuit.
We are interested in measuring any change to this load.
So this load can be something, say, you set up the circuit.
And this load can be depending on temperature or pressure
for any change.
So you have changed the Ris.
It's a modified version.
Like if x is equal to 0, you have the original setting.
But the temperature or pressure got high.
So this measurement, the impedance
will change by x percentage.
So this circuit, I want to take that out.
You can just directly compute what will be the v out.
v out is a function of x.
You set it up, you have v out.
x changed, v out will be changed a little bit.
So how you compute v out?
So this is v out equal to v in.
So you have this factor that you can
solve using the circuit analysis method.
Even complicated network, we can solve.
This simple thing, just the voltage divider,
we can certainly solve.
This is a relationship.
And the signal, that means you have a certain value v out
as a background.
The x is usually very small.
So you have a small change on top of v out here.
And I said, when you have a small signal,
you want to magnify.
But if you have some background signal there,
say you have 5 volts, then the signal change
has a magnitude, say, 1 millivolts.
You want to magnify, say, 1 million times.
Then the background signal, 5 volts,
will become 5 million volts.
This is too high.
So ideally, we want to remove the background component,
only detect the change.
So this is for later stages, so that we can magnify the signal.
So this is achieved by Wheatstone bridge.
So set up this way.
You're like R1 divided by R2, divided by R4, and R2
divided by R3.
If they have the same result, then the voltage drop
is set at this point.
The potential is equal to the potential at this point.
So if you measure v out between these equi-spatial points,
the current or voltage will be 0.
But if later on, this R3 is changed by x percentage,
then this balance will be broken.
So the v out will reflect.
So any non-zero value will directly
reflect this relative change.
So the small signal will have a zero background component.
Then we can magnify it with very high here.
Because this signal is very small,
and suppose we have a good magnifier,

we can magnify the signal, say, a million times.
That's still a reasonable signal.
It wouldn't give us a very high voltage risk.
So this is the essential idea of why we need the bridge.
This only measures one component.
Later on, we'll change.
Say this could be a sensor.
See the beam in the building.
If the beam is under high pressure,
then this impedance may be changed.
It's just an example using resistance.
The impedance could be capacitor.
If you price capacitor, the distance between two places
a little deeper, the capacitance will change.
So this bridge idea explained in terms of a resistor.
But you can consider each component
could be anything, like capacitor, inductor, resistor.
So the idea is just half of this.
Now mode measurement.
So normally, you have zero signal.
Any non-zero signal is small, you can magnify.
And the reflex mode is convenient and cheaper,
but not very sensitive.
So this is the idea about this stone bridge.
And by the way, if you can interconnect these components
arbitrarily, say, you can switch R1 and R3
or you just make a link in the connection arbitrarily.
So you can measure this arbitrary interconnection.
And I think you can solve the equation
so that you can not only monitor this one,
you may be able to monitor all of them.
Suppose you have a capability, just link them arbitrarily.
So this one, I still didn't want to work out the solution.
But halfway, he didn't finish.
But I think there's still a possibility
we can improve this wheat stone design.
Anyway, just some random comments
in case some of you are interested.
So so far, you see capacitor, inductor, and the resistor,
all can be governed by generalized almost law.
V equal to Z times I. So this is a linear thing.
And we also have nonlinear components,
just like a system, a linear system, nonlinear system.
So most important fundamental element called this diode.
So the relationship is shown here.
So the component has a directional preference.
So one way, if you just look at this part,
pretty much like a resistor.
So you have a higher voltage, then you have higher current.
The proportional factor is just the slope, is the resistance.
But go other direction.
The resistance is so big, so even you
have a very big voltage applied across the components,
the current is still very small.
That means the resistance is very big.
If voltage is too big, the whole component
will be destroyed.
So this is NP-type diode.
So you have this nonlinear relationship.
You have two of them combined together.
You have the famous transistor.
The transistor has an effect.
You have a small signal here.
On the other port, the output port,

you have the same shape of change.
But the amplitude is dramatically magnified.
So this transistor has a magnifying effect.
So previously, I said the need, small signal
you want to magnify, so transistor can do that.
This is just two dials put together.
So this is one dial, this is another one.
So as a result, this current can control
the current going this way.
The working principle really involves quantum mechanics.
There's a whole lot of different areas.
But just to show you, you have this nonlinear relationship
that can be made into an amplifier.
Transistor is an amplifier.
And the transistor, multiple transistors,
can be combined.
Nonlinear effect can be utilized.
So you can implement a logical operation.
So this logical operation, you have voltage level.
One, one, then output is one.
Both need to be one, so you have one.
This is the end node.
As long as you have one port is low,
then output will be low.
And only if both input port, input variables are high voltage,
then output will be high voltage.
So this is just electrical components
to represent a logical operation and the operation.
They're all just two input variable, A and B.
So if and only if one of them or both of them are high,
output will be high.
So all these things can be implemented
using semiconductor components shown here.
You can do a logical operation.
The logical operation can be designed,
can be grouped together to do addition, multiplication.
And along this way, that's our digital computer.
So just say we have a digital computer built
using nonlinear components.
That's why we need not only linear system,
but also nonlinear system.
And related to what we just discussed,
many, many transistors are put together
and can achieve the requirement.
Do you still remember I have three color arrows?
And I want input resistance is high.
On the other hand, I say I want to output resistance is low.
I want the magnification factor is high.
So operational amplifier, that's exactly what I explained to you.
So this is very important.
The whole circuitry filled with nonlinear components.
That is transistors.
And this look complicated, but we have a simple model.
So here, you have an input voltage.
The input voltage, so you have an input signal, small signal,
V in.
Then you have big gain factor, G.
So output V out is G, very big magnification factor.
G times V in is V out.
So this is just a simple model.
That satisfy all I said before.
So I want magnification factor or the gain is very big.
So just the mathematical simplification.
I say this is infinitely big.

Then the input impedance.
So I have a good signal drop from the measurement point of view.
So this is very high.
But once you do the measurement, this is kind of like a resistance or impedance converter.
So you drop a signal.
So you drop a signal, you need this impedance high.
Then you turn around, you take a signal out.
You relate the signal to next level.
Then the impedance from output point of view is very small.
So this is low.
So these three colors here, and I required before.
All the requirement can be satisfied by this so-called operational amplifier.
So this is a very cool thing.
And we don't have a lot of time looking into detail.
But just remember these idealized characteristics.
When you analyze circuit with operational amplifier, so this operational amplifier is a component, but not like the components we mentioned before.
Resistance, resistor, capacitor, inductor.
And the component itself is passive.
No electrical power inside.
But operational amplifier, inside this, you see the voltage Vs plus, Vs minus.
So inside the components, there are some electrical power to make it work.
So this is an active component.
But even if it is an active component, KVL, KCL.
So for any nodes, all the current coming in must equal to sum of outward flux.
And all you go around a loop, and the total voltage drop must be 0.
That still holds.
And particularly, specifically for this operational amplifier, we have two rules to perform signal analysis.
So one is that the two input terminals are at the same voltage.
This voltage drop is very small, because the magnification factor, this g, is very big.
So any small thing will be magnified, presented to the output port significantly.
So because this voltage drop between the two ports are small, so you think potential here and potential here are both 0.
This is one thing.
There's two input terminals at the same voltage.
The second, no current flow into either of the input terminals.
So current cannot flow into this much.
Because the resistor impedance is high, you've got any definite number will make this voltage drop very big.
And I already told you, the voltage drop across this must be small.
So these are two rules to analyze operational amplifier.
Just to show you one example.
So you understand the basic idea.
That's OK.
So you have this operational amplifier.
So you have Vi.
What is Vi?
Just simple like this.
So we can do analysis.
First, we say Vi, Vi out is related by this R1,

it's Ri, Ri, Ri, feedback, I think.
So I explained it to you.
So this current going this way must go this way.
No current can go into this input terminal.
So this is first rule.
So you've got this one.
This voltage is V out.
So the voltage across this is equal to 0,
because the potential here is 0.
0 minus Vo divided by Ri equal to Vi minus 0,
because this is 0 divided by Ri.
And it just changes a little bit.
So you have the relationship like this.
So just think about this.
How you do analysis.
I hope you will understand these two rules.
And if you make the current big into this,
just a little current into this input port,
you have a significant voltage drop.
This will be magnified greatly.
So this Vo will become very big, because this operational
amplifier has a big magnification factor.
So while this is very big, so it will resist the current
into this loop.
And this will make the voltage drop here.
The voltage level is high.
So it will against the current going through this way.
So this is dynamic interplay, as long
as you understand these idealized properties.
And you can understand why you have these two rules.
And otherwise, the whole system cannot be balanced out.
And there are other modes to do operational amplifier
connections.
So shown here.
So all these can be similarly analyzed.
And you click here.
All the details are in the website.
So you just read the pages if you are interested.
But here, really, I emphasize the fundamental concept.
And you do not need to worry about too much details.
So this is something that we need to know.
And as long as you understand this example,
I would consider it sufficient.
So that's how far we want to reach.
We don't want to go into too much detail.
Nonlinear components, all the fancy connections,
that's beyond the scope of this class.
We can do measurement with operational amplifier.
You can do the impedance matching.
So you're looking into this input port.
The impedance is very big.
So you've got a signal captured, this part.
But when you take a signal out, you look into this way.
From the output port, the impedance is very small.
So you've got both sides.
And the contradicting requirement I mentioned
is really, really optimized based on these components.
So so much for the electrical circuit analysis.
So remember the VI, the voltage current relationship,
for resistor, for capacitor, for inductor.
Then you link them together into a network.
You have KVL, KCL, KVL.
With KCL, KVL, I explained using one slide.
You have enough independent equations

to solve them uniquely.
And this is not only good for DC circuit,
also good for AC circuit, because AC is DC in this class.
And you put in the algebraic equation,
so just solve the system equation.
You do not need to solve differential equation.
And certainly, we only talk about steady response.
The system in the steady state, so you
can use a phasor notation.
If you just deal with transient status,
you still transient state.
You just add a signal.
All of a sudden, then there are some temporal response
that need to be solved using differential equation still.
But if time is long enough, everything is stabilized,
you can just solve the algebraic equation.
And this is so much about the circuit.
The circuit can be applied for different purposes,
like for signal sensing, and you get a signal from sensor.
You can use the circuit to do filtering.
So two examples mainly.
So you see those examples as application of Fourier analysis
and some real signal interface for medical imaging.
You really need to detect the signal.
So the data acquisition involves signal detection.
So these two applications are both very important.
So you need to understand.
So next, let me say a little bit.
It's not in the previous syllabus,
but I think it would be nice if we move on
to say a little bit about the neurological network
and the artificial neurological neural network.
Artificial neural network is a mainstream method
for machine learning.
As it goes back, more than two years ago,
we organized a symposium in this meeting.
We talked about X-ray imaging for medication.
It's AAAS, and if you know the magazine, Science Magazine,
Science Magazine is an official magazine of AAAS,
something like American Advancement Science,
something just to double check.
But anyway, this is a very important meeting.
All the areas, medicine, education, engineering,
mathematics, physics, they all come to this meeting,
just for good education, pretty much.
And then we finish our symposium.
The meeting consists of many symposiums.
And then we finish our symposium.
Next day, I have nothing to do, so I
attended this symposium called the Technology
of Artificial Intelligence.
So I heard several exciting presentations.
So I got convinced that now the artificial intelligence
enters the revolutionary stage, and that everything
becomes so exciting.
So I just got the idea, talk about IT technology.
Earlier days, we say industrial technology,
and then the IT information technology.
So with machine learning or artificial intelligence,
so we talk about the intelligence revolution.
So make a machine very smart and do things
that we cannot do before.
So in terms of scientific research,
and then you can do experimental observation.

You can build a theory, and then you
can derive things once you have a theory.
And then also, you can use a powerful computer
to do simulation based on theory.
And then now we have a lot of data, so big data.
This is machine learning.
We can extract information, feature, relationship
from data directly.
So this is so-called scientific paradigm, science paradigms.
So empirical is the first one.
Then the big data, machine learning,
is called a false paradigm.
So this is the totally new thinking.
And this is a picture, biological neuron.
And some of you may be very familiar.
And if you're interested, you can certainly
do Google search.
And a simpler version, so you have a biological stimuli
taken by dendrites.
Once the strength of stimulation is strong enough,
electrical signal will be generated
and passed along the nerve to the next stage
and excite downstream neurons.
So this is a human neurological system,
and a very simple version.
And let me just say this, further simplified,
simplest version.
So input port kind of multiple branches called dendrites.
So you have this cell body called a soma.
Then the signal, once generated by the neuron,
will be passed downstream along axon.
So output interface through synapses.
So this is a neurological thing.
And we do mathematical modeling.
We say all these input port can be modeled as a variable.
And the strength of the branches are not all the same.
So we have a weighting factor.
So you have a smaller weighting factor,
means this variable doesn't play a significant role.
So all these stimuli added together.
So we do summation here.
So you have multiple variable weighted together
and added together.
This is what we learned.
This is called inner product.
We learned before, so inner product is very important.
This is linear operation.
You've got everything summed together and not finished yet.
If you only have small stimulation,
the neuron doesn't care.
So this gives certain stability.
The small noise, usually you don't care.
Just small things, just your walk in the supermarket,
you'll have a small input.
Other people push you or just pass by.
And by incident, you don't care.
But if someone really kicks you, they're strong enough,
you will fight back.
So this is pretty much like a neuron.
When the input is high enough, you
have a nonlinear transformation.
Input is small, it's zero.
When input is high, it gives high response.
So this is just a neurological counterpart in computer.

So the neuron, or called perceptron, models as this.
So multiple variables weighted, you perform.
This part is inner product added together.
You can add an offset.
Just think that you have input 1, weighting is b.
Your input is b, weight is 1.
So you've got this offset, like a DC component.
All these things are summed together.
It's an inner product.
This inner product is precise by a nonlinear function sigma
as an output.
So this is just a single neuron.
Single neuron is not that complicated, as I explained.
So the nonlinear activation function can be sigmoid,
and it can be many things, like this.
Even this linear function is not a purely linear function.
It can go this way, like a linear unit.
So when the input is zero or negative, nothing happens.
This is the output zero.
But once you have positive input,
it will give you proportionally high output.
This is a nonlinear input function, not directly linear.
Otherwise, it's just a linear component.
Then you need to use the neuron as a building block.
Previously, we say we use the sinusoidal as a building block,
use delta function as a building block.
And for electrical system, building blocks
are resistor, capacitor, inductor,
and even the operational amplifier.
But for neurological system, artificial neurological system,
the building block is this artificial neuron.
So all kinds of neurons, but it can
face different weighting factor.
Even different activation function
can be put together into many layers
to make things more and more complicated.
So the single neuron cannot do much.
But for this kind of neural network,
once you put more and more layers,
it can do amazing things, like an auto driving car.
You have many image recognizer, recognize traffic sign,
and so on.
So lower level, the input is, say, picture.
The next level gives you some small point or add response.
And the later stages get the feature combined
at a higher and higher level.
Eventually, it will match to the knowledge
if the picture is a car, or dog, or cat, so multiple levels.
This will show you input layer, output layer.
Between input and output layers, you
could have hundreds of hidden layers.
You can link this way.
This is feed forward thing.
And you have big data.
So it means you have many, many images.
You want the neural network to tell you
if the image is a dog or cat.
So you have a training data set.
With that training data set, so you randomly
put the weighting factors.
The network wouldn't do well.
So it just randomly put number.
So you say it's a dog.
So this value is high, then you say dog.

This value is high, you say cat.
Initially, you randomly put the weighting factors,
the neurons, activation functions.
So you just get a random thing.
On the output port, you know how you made a lot of mistakes.
You call dog, cat, you call car, breeze, it's all messed up.
So you have an error here.
Then this error will be feedback.
So you try to change each of the parameters
so that the error can be reduced.
This is called a training process.
You need to do this multiple times.
Just try an error with big data, a lot of data to train.
And given enough iteration, the training done thoroughly enough,
and the network will work.
Just give you good results.
That's a very amazing thing.
This is feed-forward network.
You can consider many kinds of connections.
So this is a whole lot of different network topology.
And not all the configurations are equally useful.
You need to have some engineering inside,
like you put a layer by layer feed-forward.
That's a kind of a multi-resolution analysis.
So many things, and you can use complex network
because you think certain things like AC circuit,
like electrical magnetic waves, you better
use a phasor complex analysis, phasor notation.
So you feed the network all phasors,
and then you use a complex number as a weight.
So this is a very active area.
How do you do the optimization?
As I say, initially, your network
wouldn't give you good performance.
You change the parameter.
So you change the parameter.
You change the w, just the vector, all the parameters.
It can be many parameters.
You keep changing so that on the output port, the loss
function, the difference between what current network output
and what you want network to output.
This error, this frequency, gets smaller and smaller.
So network is being trained, eventually converge,
give you good results.
And MATLAB now has a new toolbox called the Machine Learning
Toolbox, the parameter optimization.
Once you define a loss function, the discrepancy
between ground truth and the network output,
by changing the parameter, so you can just
reduce the loss function and just get a smaller, smaller
value, eventually converge to a good solution.
This is something illustrated in this YouTube movie.
So just for those who are interested, you read it.
Otherwise, this is roughly understand
we have a network called neural network,
artificial neural network.
And through training, it can do amazing things.
It just depends on how much you want to know.
This got very hard.
And as an imaging scientist, we say
this machine learning method can be used for medical imaging.
The conventional way is to have images in the first place.
But we do medical imaging.
Images is the result. So we want to generate images

using machine learning method.
So this is an example.
See, with neural network, this now is an image.
And we can just clean out the image, beat other method.
So this is a very promising technology.
We get prior knowledge from a huge data set.
Like now in hospital clinic, you scan patient every day,
many, many images.
Then you just throw away.
But now we say keep all these images
and we can extract knowledge.
Using these images as a training data set.
And we can do better job.
So there's so much for today.
And for your homework due today, and this is just an example.
I wouldn't post this to IOMS, but just
let you to take a look.
So this is something summarized or key point
for the foundational part.
So you can think to derive something like this.
So you have all the relationship on one poster.
And some pictures like this one, there are multiple versions.
So I draw this to avoid copyright issue.
But anyway, in classroom, that should be no problem.
But eventually, you want to publish.
You either get permission or you better
redraw into something you like.
And for homework today, you analyze this.
So just try to see how you set up the register so that
the sensitivity is maximized.
The sensitivity defined as the first derivative of V out
with Dx.
So the x changed a little bit.
Then you want V out change very high.
That's called the sensitivity is high.
You can select RL and RS.
So actually, you make them both the same.
You get a maximum value.