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Lecture 18: MRI Physics

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Ge's Talk at RICAM, 3/30/17

Radon Transform via Machine Learning

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The Radon transform is widely used for tomographic imaging. Due to the penetrating power, fine resolution, complementary contrast mechanisms, high-speed, and cost-effectiveness of the x-ray technology, computed tomography (CT) is one of the earliest and most popular imaging modalities in biomedical and other fields. The recent advancement in deep learning, or machine learning in general, promises to perform and invert the Radon transform in innovative fashions. This direction might lead to intelligent utilization of domain knowledge from big data, innovative approaches for image reconstruction, and superior performance in important applications. In addition to a general perspective (*Ge Wang, "A Perspective on Deep Imaging", IEEE Access 4: 8914 – 8924, 2016; http://ieeexplore.ieee.org/document/7733110*), some "deep imaging" results at our Biomedical Imaging Center will be also discussed.

MRI Physics

- **Preview & Review**
- **Physical Foundation**

  Magnetic Moment (Quantum Mechanics)

  Magnetization (Classic Model)

  Precession
- **Signal Generation**

  RF Perturbation **(**Rotating Frame)

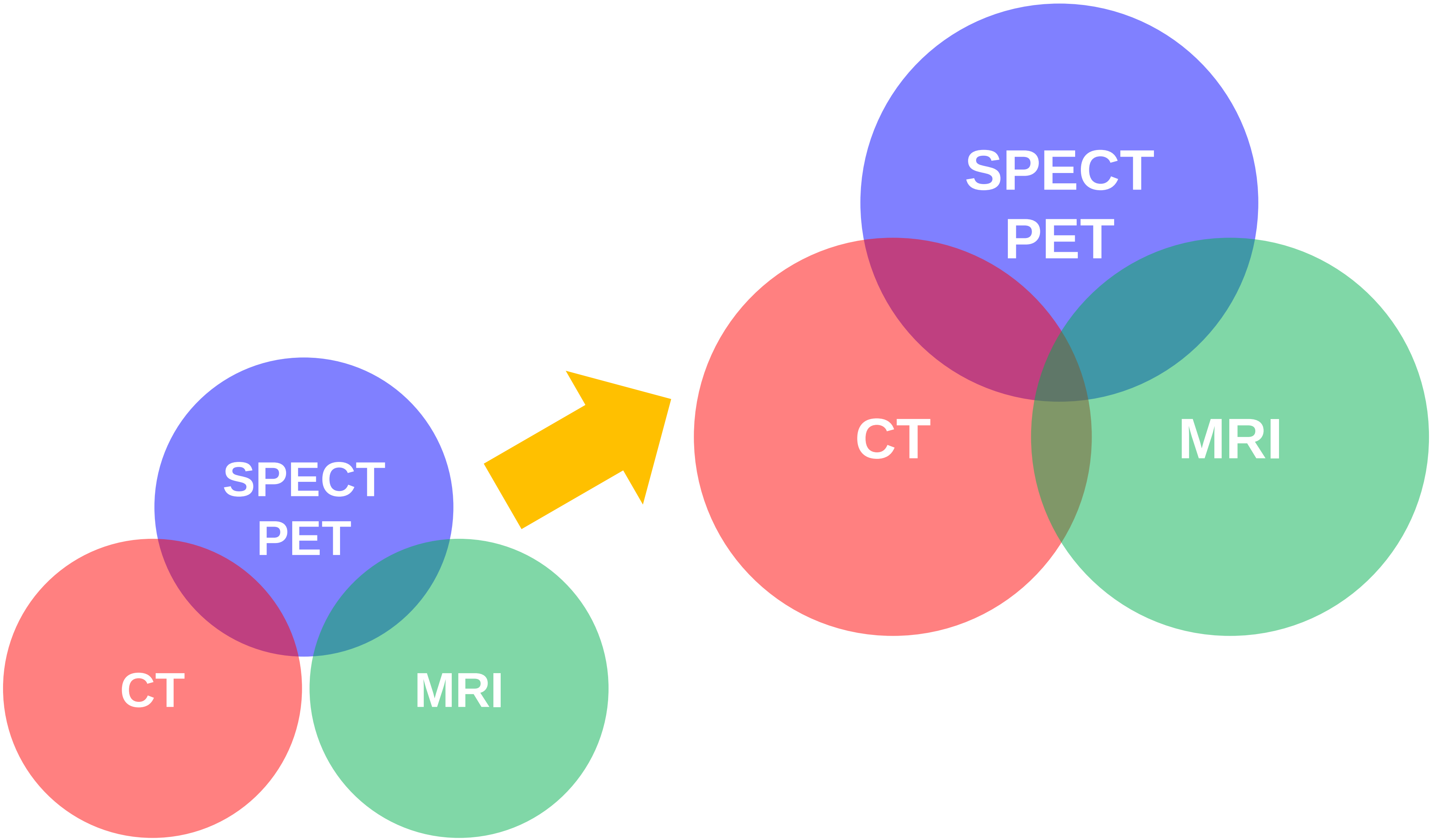
  Bloch Equation (Parameters T1 & T2)

  FID Detection
- **Signal Decay**

  T1 & T2 Mechanism

  Inversion Recovery & Spin Echo

Anatomical, Functional, & Hybrid



MRI Milestones

MRI Measures Magnetic Resonance Specific to Soft Tissue Types (Anatomical) & Blood Oxygenation Level (Functional)

1946: Nuclear Magnetic Resonance (NMR)
Bloch (Stanford) and Purcell (Harvard)
1952 Nobel Prize in Physics

1973: Magnetic Resonance Imaging
Lauterbur (Stony Brook University)
2003 Nobel Prize in Medicine

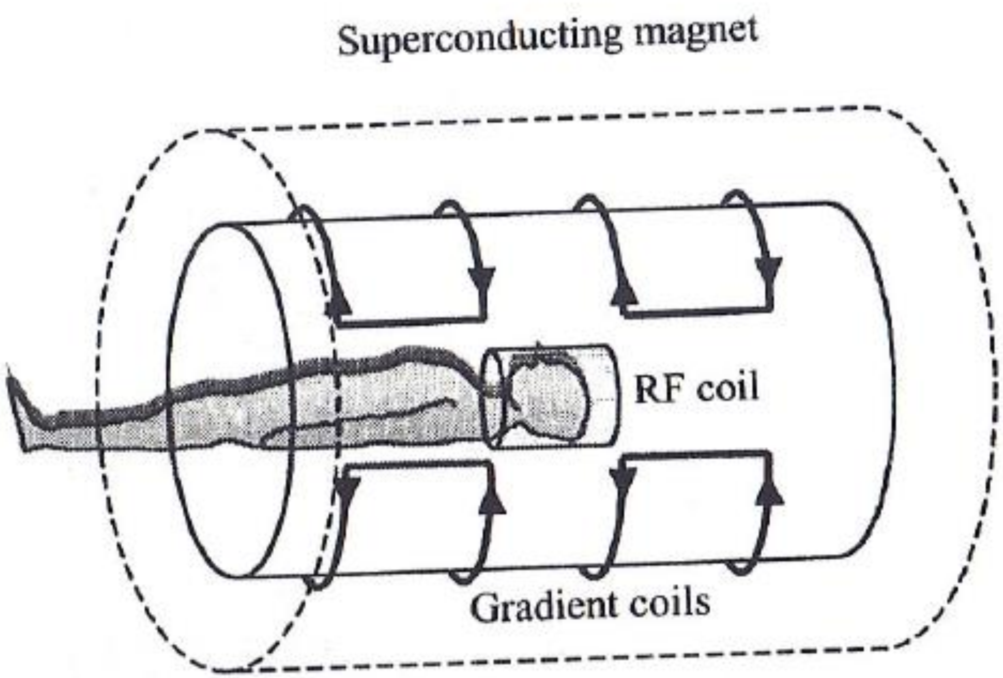
Late 1970's: Human MRI images

Early 1980's: Commercial MRI System

1993: Functional MRI in Human

Now: Brain Initiatives

Basic Idea



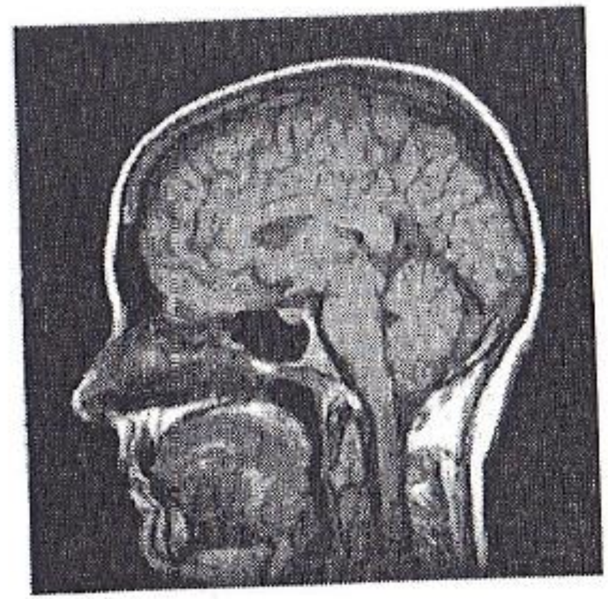
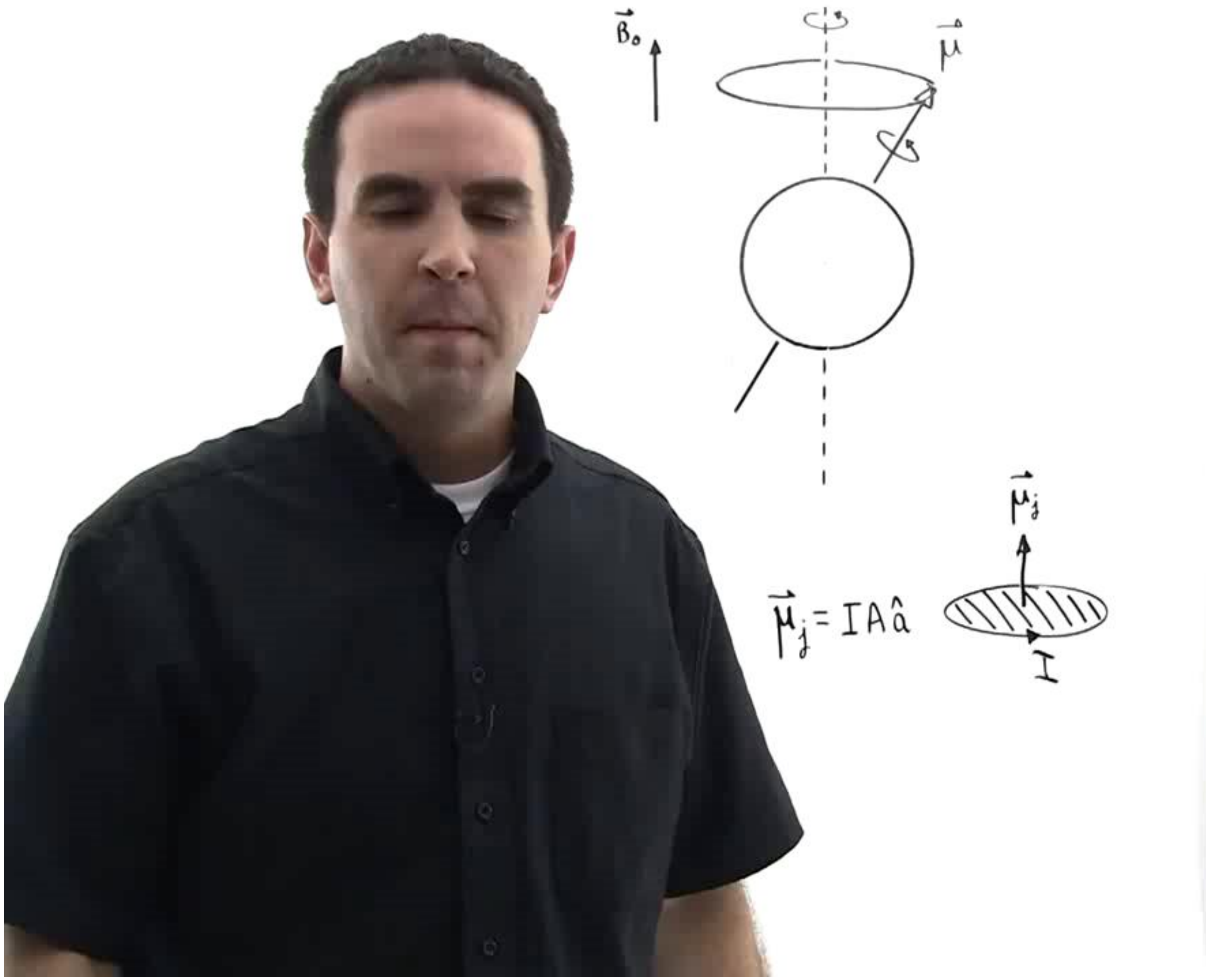
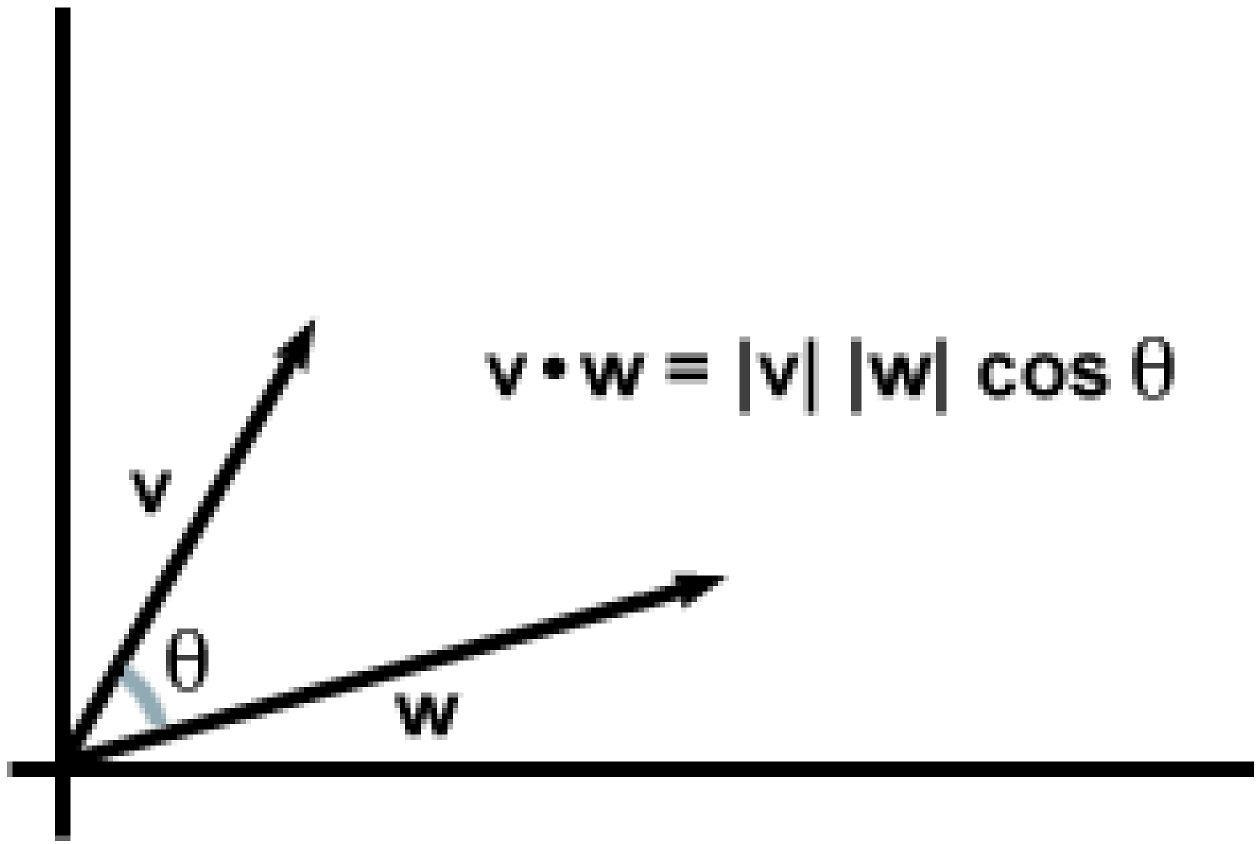


FIGURE 4.1. (Left) The instrumentation involved in MRI consists of a superconducting magnet, three sets of magnetic field gradients (only one is shown), and a radiofrequency coil. (Right) A single-slice MRI of the brain showing excellent soft-tissue contrast between gray and white matter and high spatial resolution.

Nice MRI Talk!



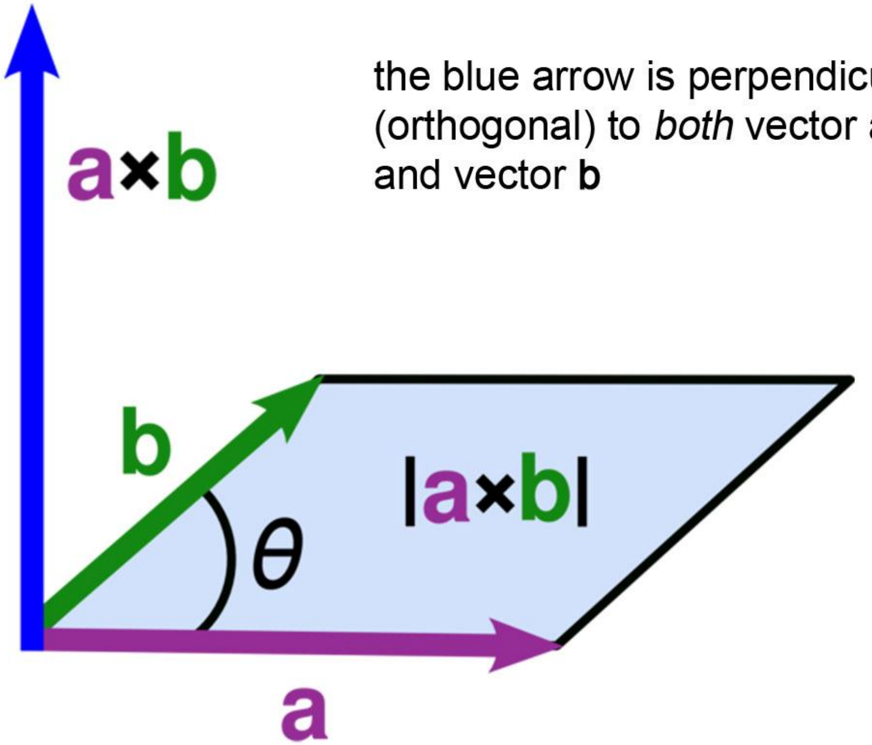
Dot Product



Cross Product

blue arrow is the resultant vector, with scalar value a times b times sine(θ), which is the area of the parallelogram in the plane of **a** and **b**

the blue arrow is perpendicular (orthogonal) to *both* vector **a** and vector **b**

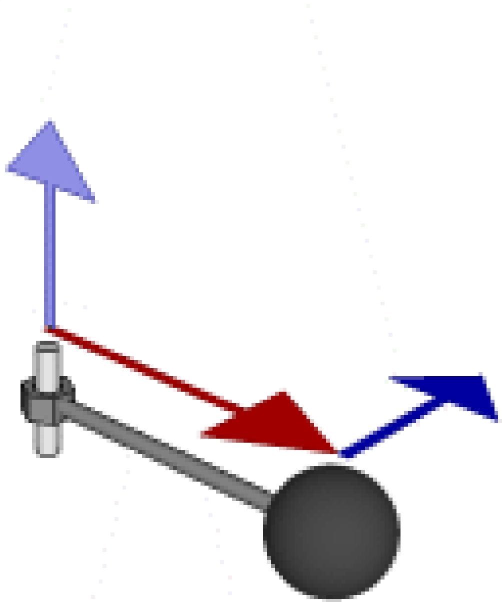


Cross-product of two vectors a and b separated by angle θ. Vertical vector is the cross-product value & direction

Torque Changes L

A Torque over Time Makes a Change in an Angular Momentum (F=ma in Disguise)

$$\frac{dL}{dt} = \tau$$

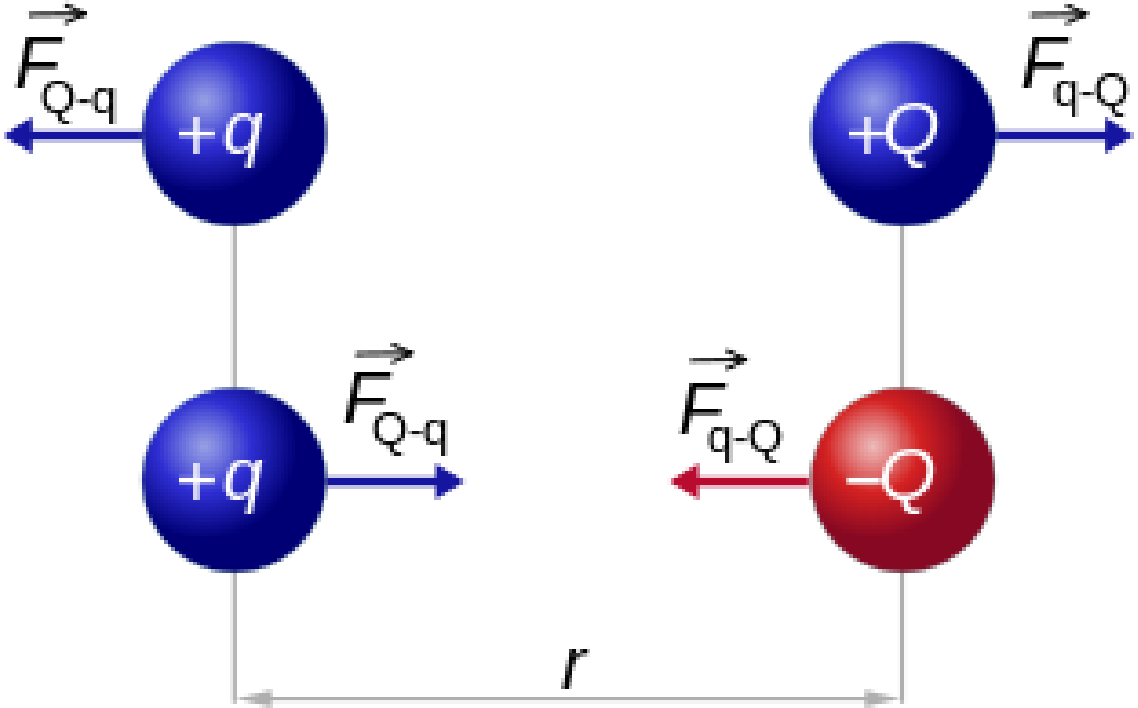


$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

Note: p=mv, dp/dt=F

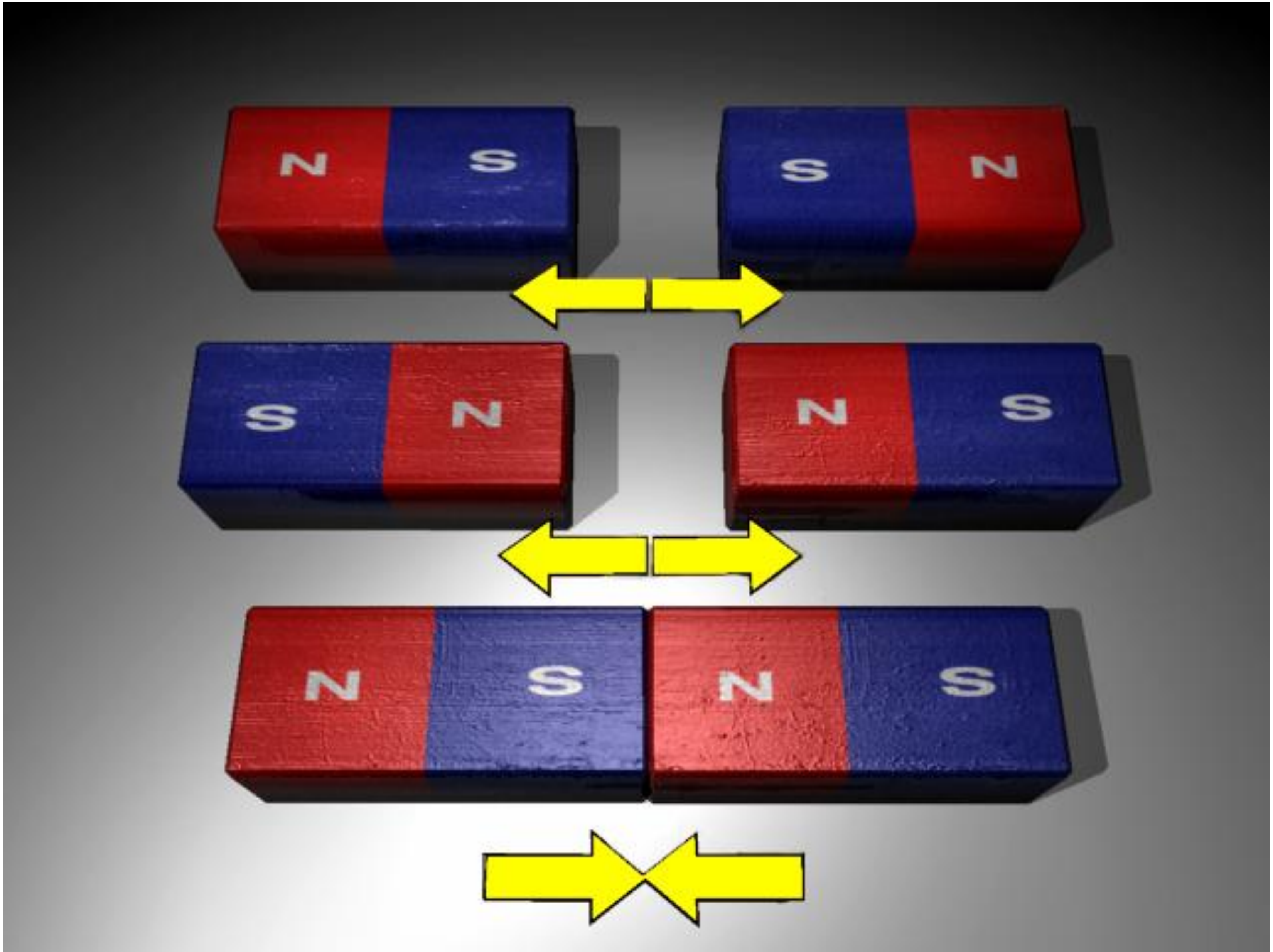
Relationship between force **F**, torque **τ**, linear momentum **p**, and angular momentum **L** in a system which has rotation constrained in one plane only (forces and moments due to gravity and friction not considered)

Electric



$$|\vec{F}_{Q\text{-}q}| = |\vec{F}_{q\text{-}Q}| = k\,\frac{|q \times Q|}{r^2}$$

Magnetic

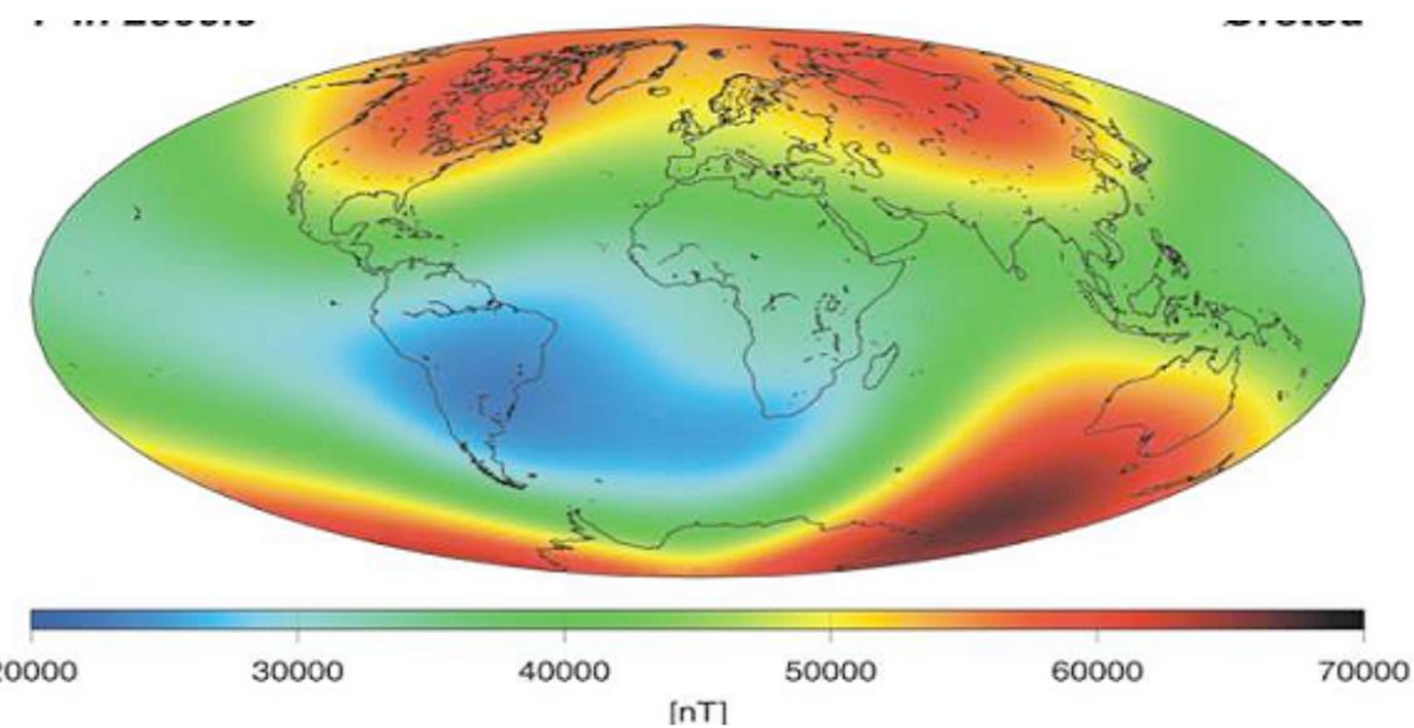


Magnetic Field

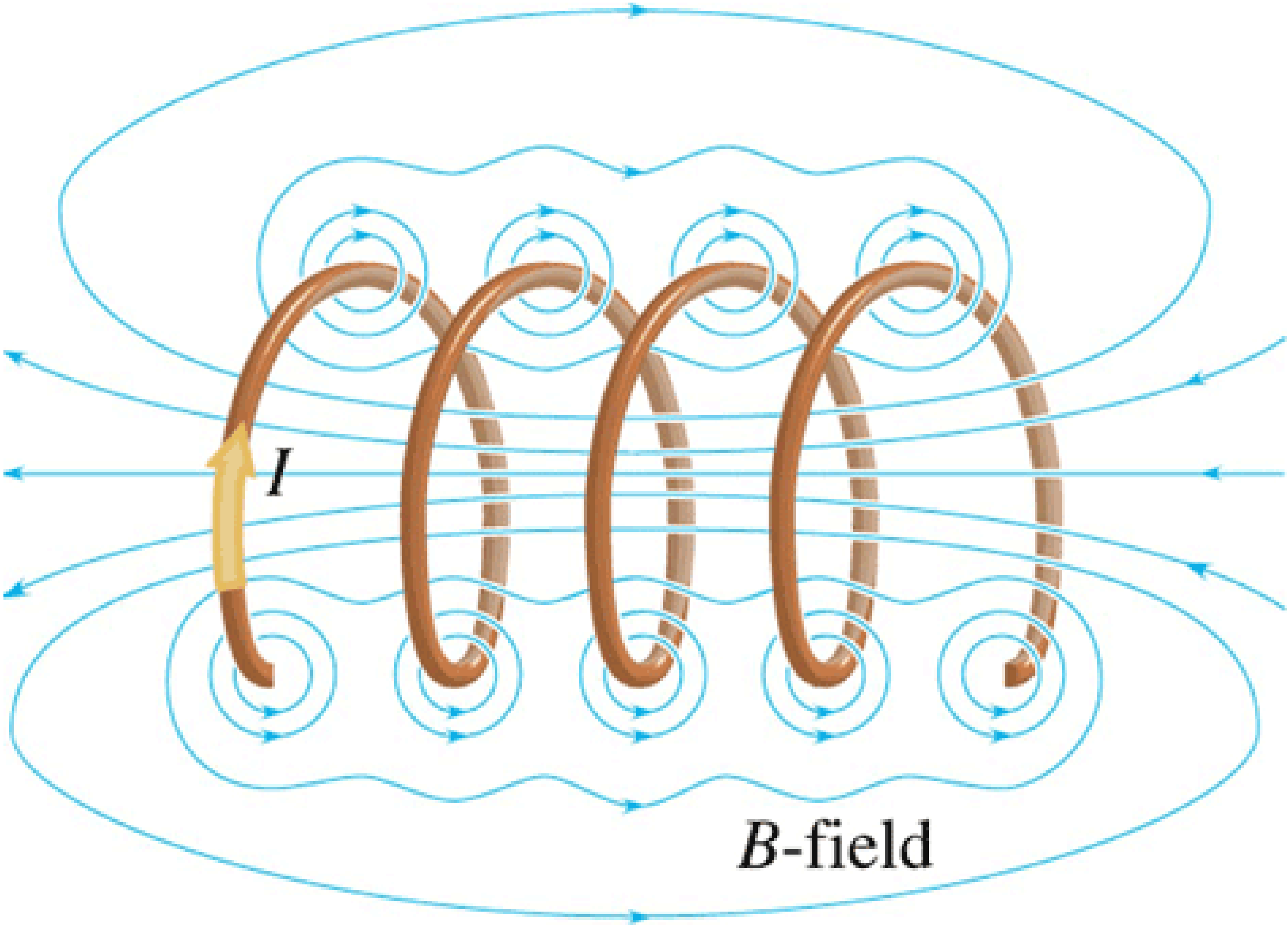
1 Tesla = 10,000 Gauss

Earth's field is about 0.5 Gauss

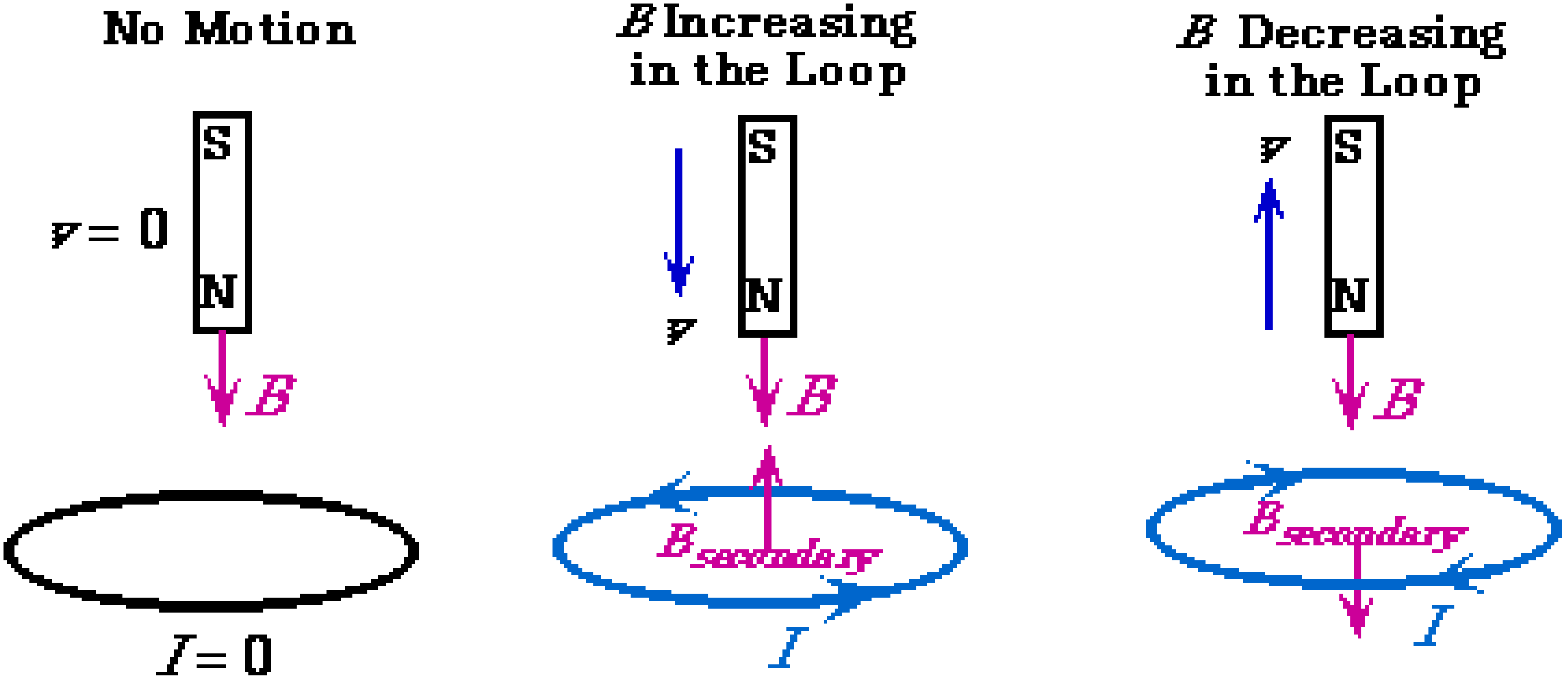
$$0.5 \text{ Gauss} = 0.5\text{x}10^{-4} \text{ T} = 50\ \mu\text{T}$$



E->M



M->E



No Motion
$\vec{v} = 0$
S
N
$\vec{B}$
$I = 0$
$\vec{B}$ Increasing in the Loop
$\vec{v}$
S
N
$\vec{B}$
$B_{secondary}$
I
$\vec{B}$ Decreasing in the Loop
$\vec{v}$
S
N
$\vec{B}$
$B_{secondary}$
I

In the beginning…

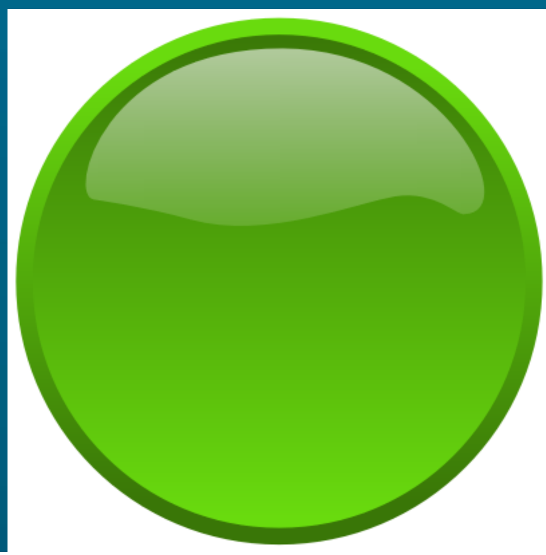
- In the beginning, God created the Heaven and the Earth …
- … and God Said:

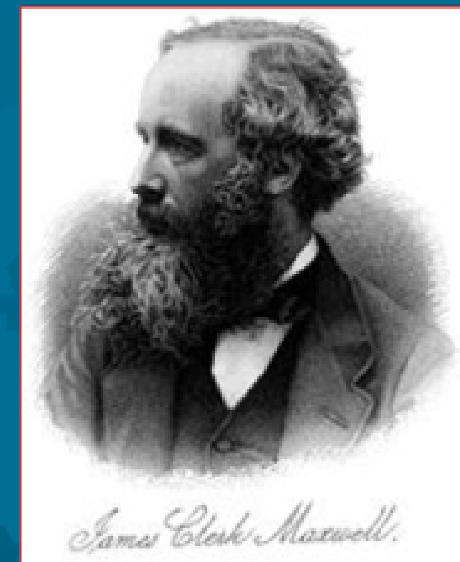
$$\nabla \bullet \vec{D} = \rho$$

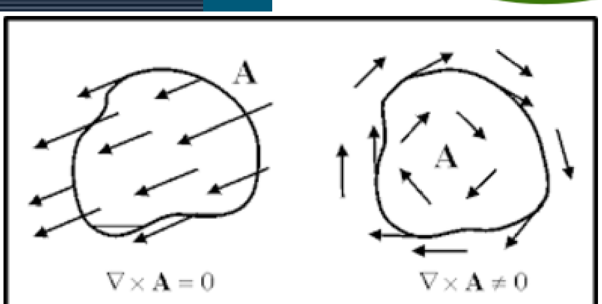
$$\nabla \bullet \vec{B} = 0$$

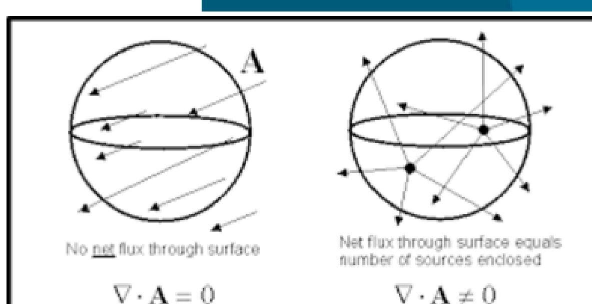
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$







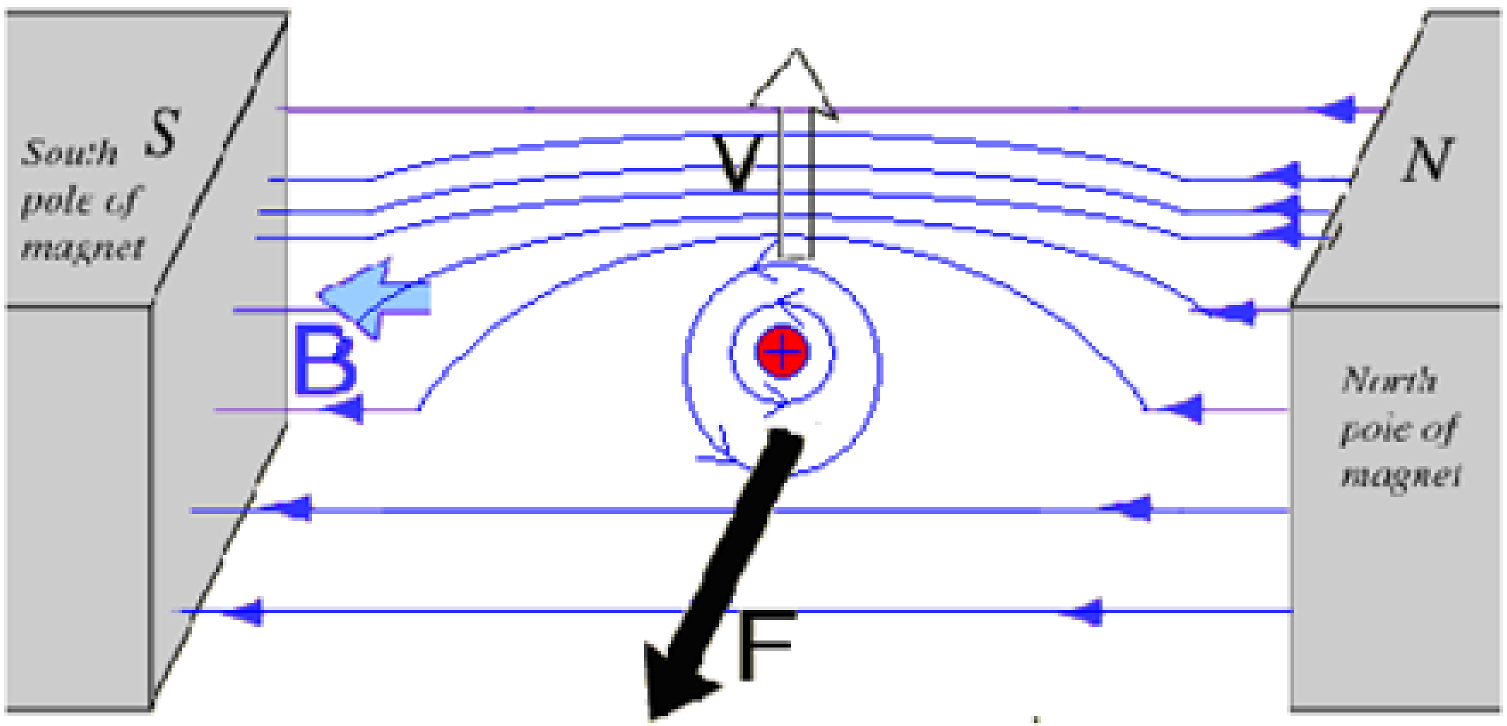


$$\nabla(\ ) = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

Lorentz Force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Electric force — *Magnetic force*



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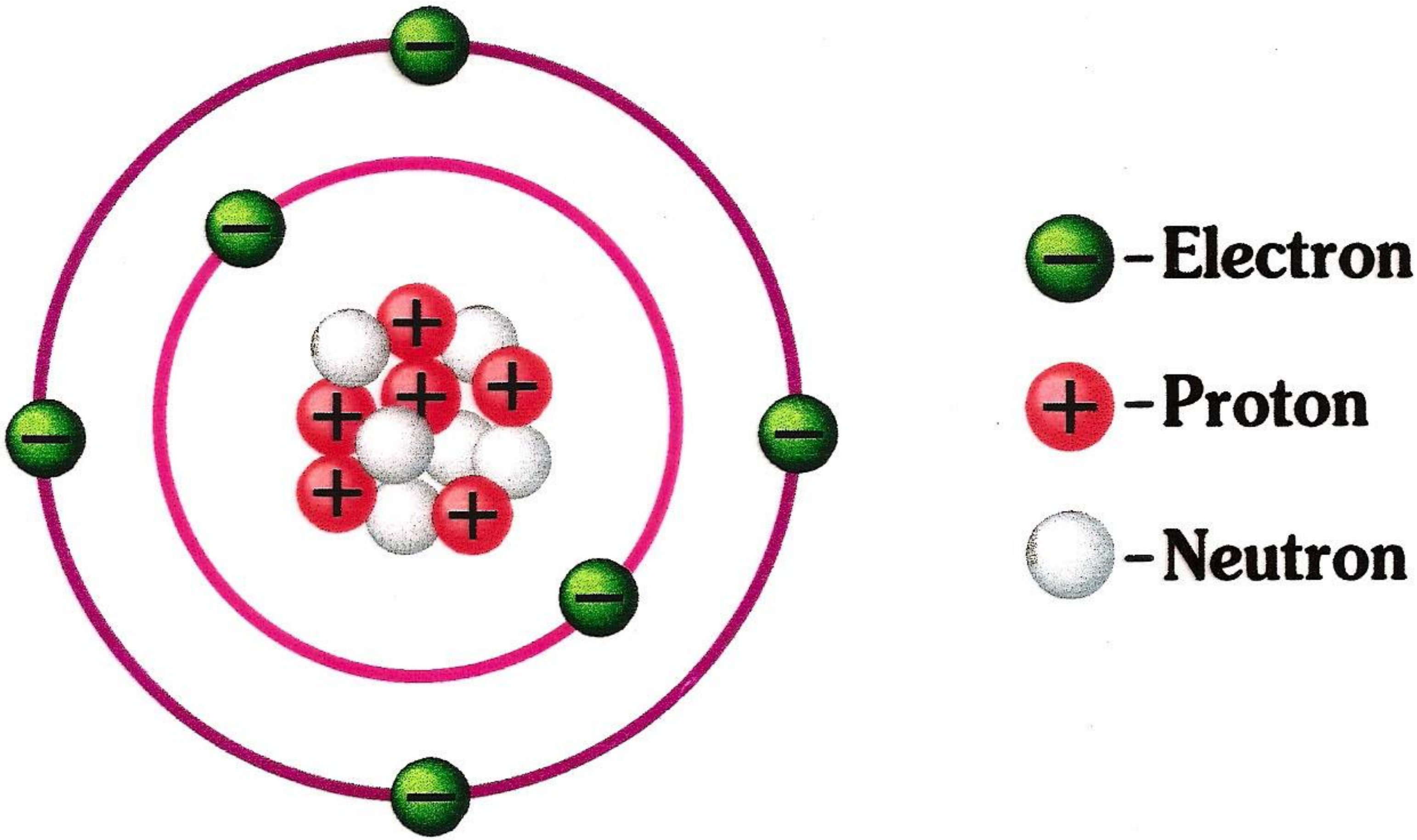
Atomic Structure



Spinning Proton

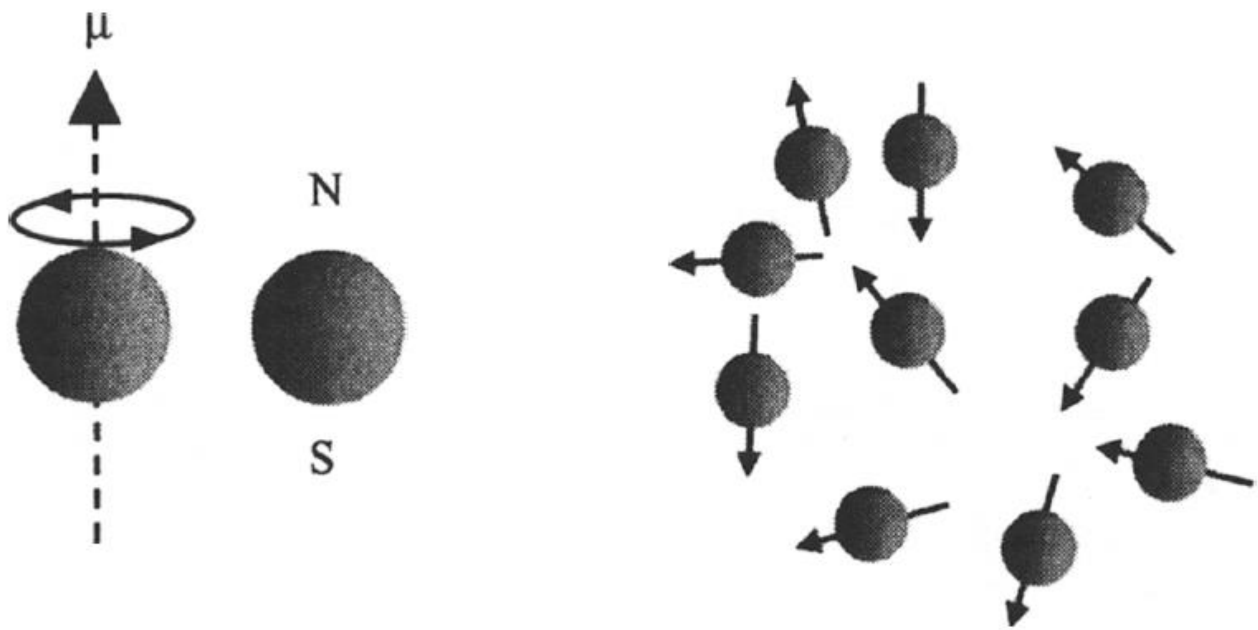


FIGURE 4.2. *(Left) A spinning proton possesses a magnetic moment μ and acts as a small magnet with a north and a south pole. (Right) In the absence of an external, imposed magnetic field, the orientations of the magnetic moments are random. There is therefore zero net magnetic moment in any given direction.*

Angular Momentum P & Magnetic Moment μ

$$|\boldsymbol{P}| = \frac{h}{2\pi}\,[I(I+1)]^{1/2} \tag{4.1}$$

the spin quantum number I

where h is Planck's constant (6.63×10^{-34} J s). The value of I depends on the number of protons and neutrons in the nucleus, and is nonzero for nuclei having an odd atomic number, an odd number of neutrons, or both. In the case of protons, the value of I is 1/2, and so the magnitude of **P** is given by

$$|\mathbf{P}| = \frac{h}{2\pi}\frac{\sqrt{3}}{2} \tag{4.2}$$

The magnitudes of the magnetic moment and the angular momentum of the proton are related by

$$|\boldsymbol{\mu}| = \gamma\,|\mathbf{P}| \tag{4.3}$$

Important Link between Mechanical & Electromagnetic Stuff

Longitudinal Component

For the proton the magnitude of the magnetic moment is therefore given by

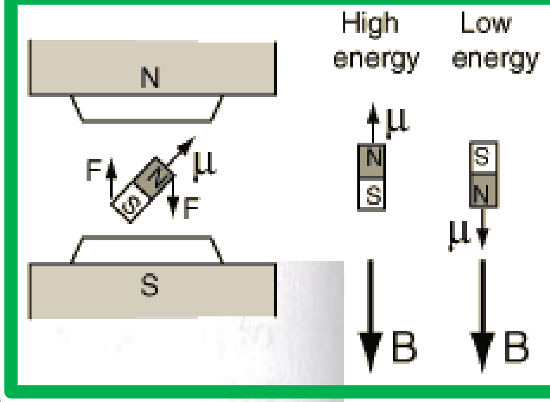
$$|\mu| = \frac{\gamma h\sqrt{3}}{4\pi} \tag{4.5}$$

The magnetic moment, being a vector, contains three components (μ_x, μ_y, and μ_z), each of which can have any value, provided that equation (4.5) is observed. However, in the presence of a strong magnetic field $\mathbf{B}_0$ the value of μ_z can only have values given by

$$\mu_z = \gamma P_z = \frac{\gamma h}{2\pi} m_I \tag{4.6}$$

where m_I is the nuclear magnetic quantum number, and can take values I, I − 1, . . . , −I. So, in the case of a proton, m_I takes two values, $+1/2$ and $-1/2$, and the corresponding values of μ_z are $\pm\gamma h/4\pi$. Because the total magnetic moment is given by equation (4.5), it is clear that the magnetic moment is oriented in a direction only partially aligned with (parallel) or against (antiparallel) the main magnetic field, as shown in Figure 4.3.

Zeeman Effect



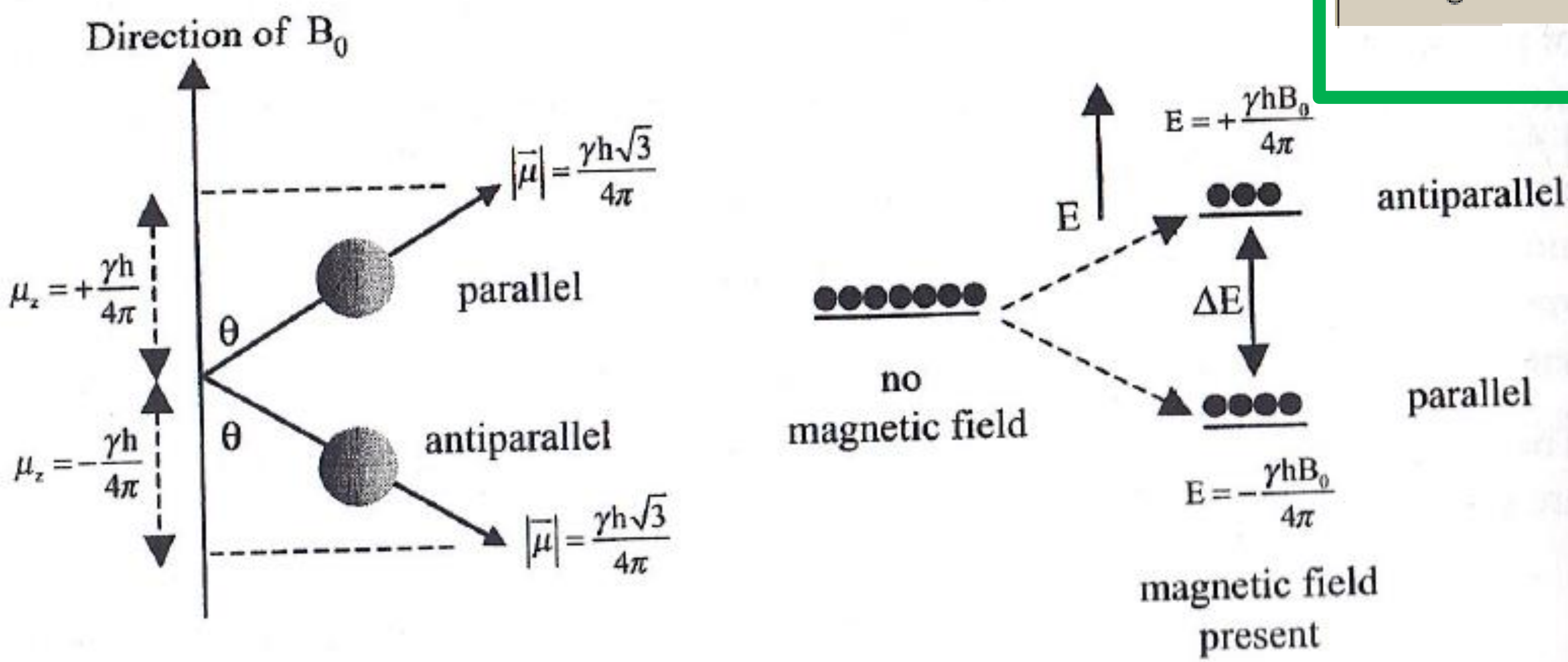


FIGURE 4.3. *(Left) The quantization of the magnitude of the z component of a proton's angular momentum means that the proton's magnetic moment has two possible physical orientations, parallel and antiparallel, with respect to the direction of the main magnetic field. The value of the angle θ is 54.7°. (Right) In the absence of an external magnetic field, there is only one energy level. When the external magnetic field is applied, Zeeman splitting results in two energy levels, with more protons occupying the lower energy level, corresponding to the proton magnetic moments being aligned parallel to the main magnetic field, than the higher energy level, corresponding to an antiparallel alignment.*

Energy Difference

$$E = -\mu_z \mathbf{B}_0 \qquad (4.7)$$

where $\mathbf{B}_0$ was defined previously as the strength of the magnetic field. So, from equations (4.6) and (4.7)

$$E = \mp \frac{\gamma h \mathbf{B}_0}{4\pi} \qquad (4.8)$$

The two possible interaction energies correspond to the protons being in the parallel configuration (E is negative, implying a lower interaction energy) and the antiparallel configuration (E is positive, a higher interaction energy). The energy difference between the two states is shown in Figure 4.3 and is given by

$$\Delta E = \frac{\gamma h \mathbf{B}_0}{2\pi} \qquad (4.9)$$

Tiny Difference

The Boltzmann equation can now be used to calculate the relative number of nuclei in each configuration:

$$\frac{N_{\text{antiparallel}}}{N_{\text{parallel}}} = \exp\left(-\frac{\Delta E}{kT}\right) = \exp\left(-\frac{\gamma h \mathbf{B}_0}{2\pi kT}\right) \tag{4.10}$$

where k is the Boltzmann coefficient, with a value of 1.38×10^{-23} J/K, and T is the temperature measured in kelvins. A first-order approximation can be made ($e^{-x} \approx 1 - x$):

$$\frac{N_{\text{antiparallel}}}{N_{\text{parallel}}} = 1 - \frac{\gamma h \mathbf{B}_0}{2\pi kT} \tag{4.11}$$

The magnitude of the MRI signal is proportional to the difference in populations between the two energy levels:

$$N_{\text{parallel}} - N_{\text{antiparallel}} = N_s \frac{\gamma h \mathbf{B}_0}{4\pi kT} \tag{4.12}$$

where N_s is the total number of protons in the body. At an operating magnetic field of 1.5 T, equation (4.12) shows that for every one million protons, there is only a

20.7. THEOREM. *Newton's 2nd Law in terms of $\vec{L}$ and $\vec{\tau}$.*

$$\frac{d}{dt}\vec{L} = \vec{\tau}_{tot} \tag{xx.1}$$

If internal torques cancel (see Theorem 19.1 for the condition that this will happen), which we will assume to be the case in this course, then

$$\frac{d}{dt}\vec{L} = \vec{\tau}_{net} \tag{xx.2}$$

where $\vec{\tau}_{net}$ is the sum of torques to due to external forces.

PROOF. $\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$. When applying $\frac{d}{dt}$, note that the product rule applies.

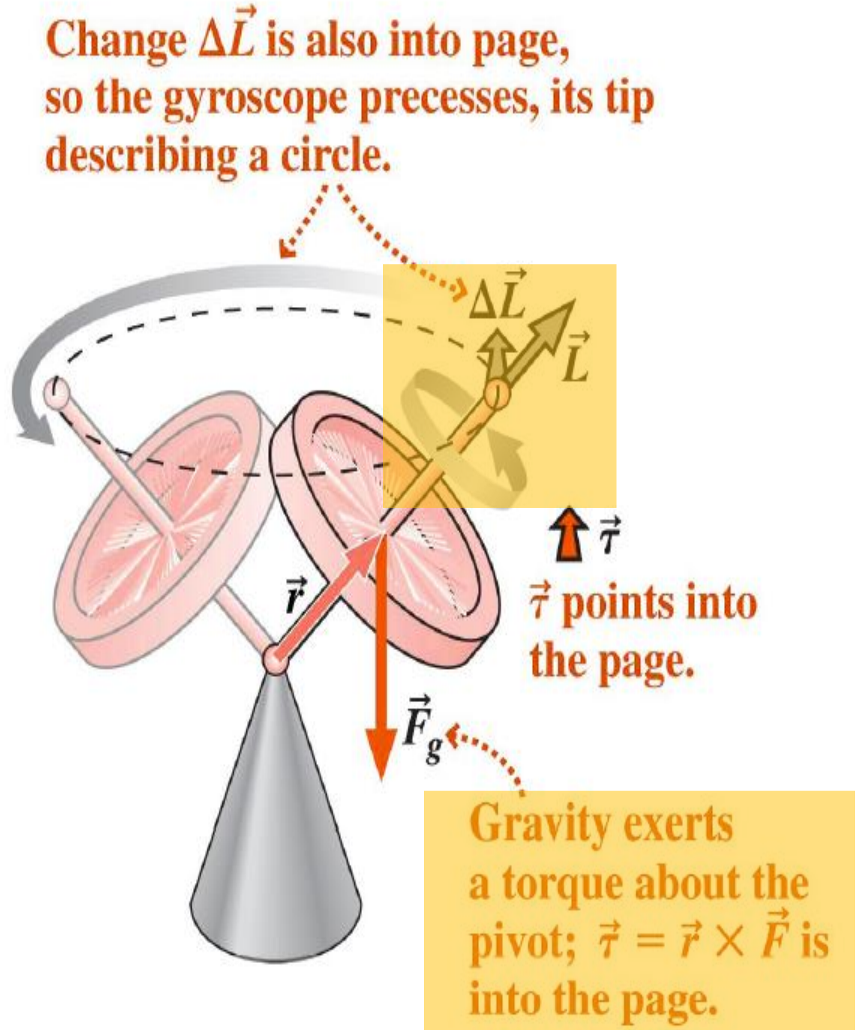
$$\frac{d}{dt}\vec{L} = \sum_i \dot{\vec{r}}_i \times \vec{p}_i + \sum_i \vec{r}_i \times \dot{\vec{p}}_i$$

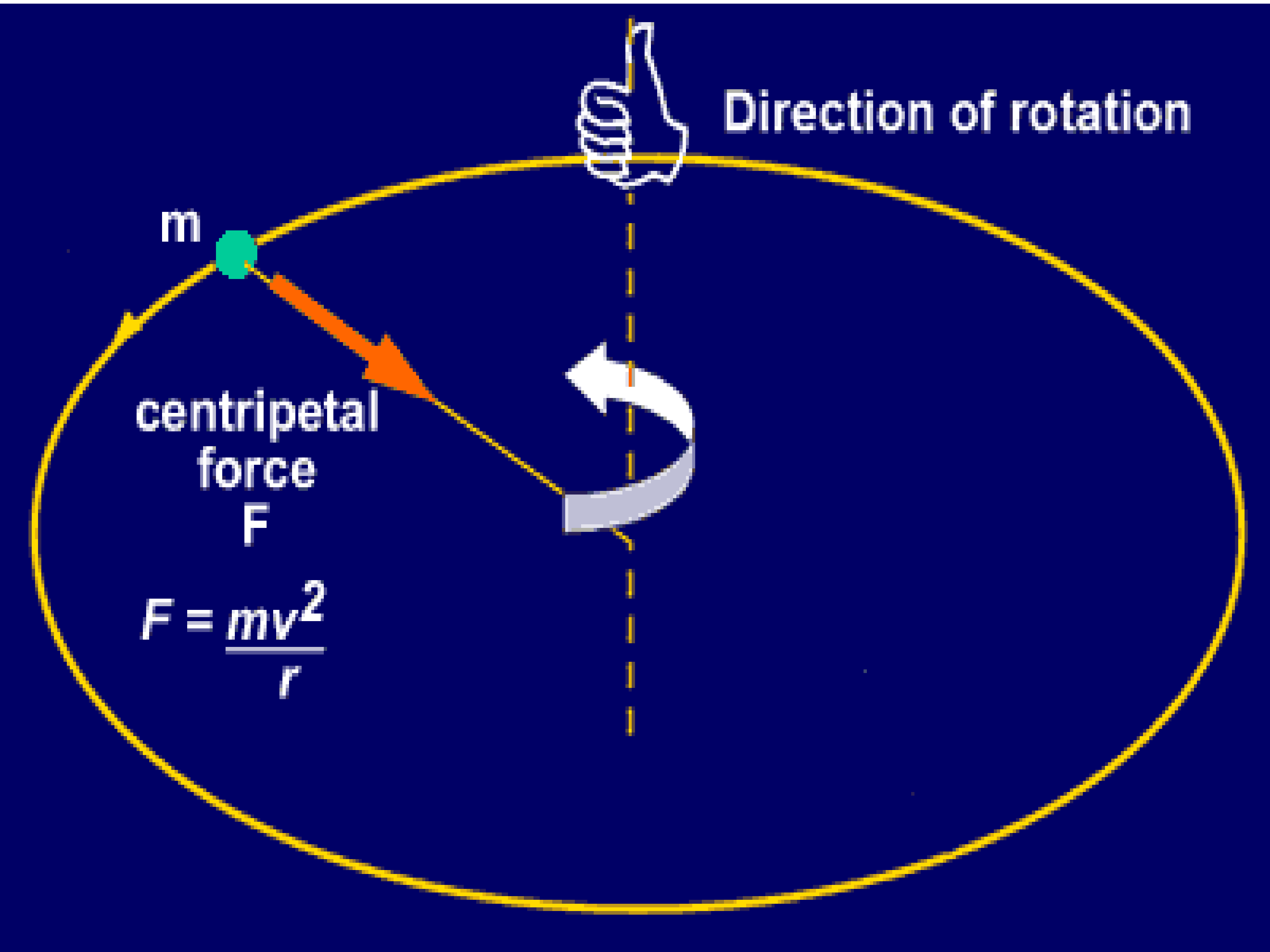
The first term is 0, since $\vec{p}_i = m_i\dot{\vec{r}}_i$ is parallel to $\dot{\vec{r}}_i$ (thus $\theta = 0$). Noting that $\dot{\vec{p}}_i = \vec{f}_i$ (*the total* force acting on the particle i) we have

$$\frac{d}{dt}\vec{L} = \sum_i \vec{r}_i \times \vec{f}_i = \sum_i \vec{\tau}_i = \vec{\tau}_{tot}$$

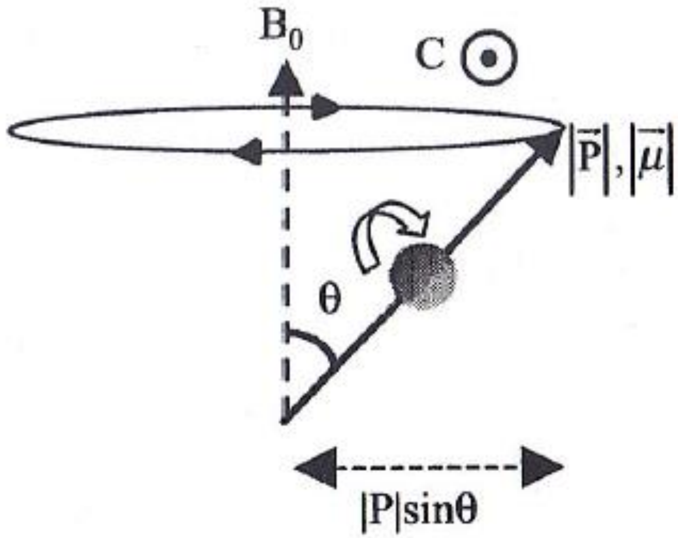
In normal cases in which internal torques cancel, $\vec{\tau}_{tot}$ can be replaced by $\vec{\tau}_{net}$, the sum of all torques due to *external* forces. □

NOTE. **Warning: $\tau_{net} = I\alpha$ and $\vec{\tau}_{net} = \frac{d}{dt}\vec{L}$. (optional advanced topic, but please keep in mind the last summarizing paragraph at the end!) The latter is more fundamental than the former, which was derived in Theorem 19.1. There are a few reaons. (1) Note that** both $\vec{\tau}$ and $\vec{L}$ are *dependent* on the choice of the origin of the coordinate system, while I


Change $\Delta\vec{L}$ is also into page, so the gyroscope precesses, its tip describing a circle.
$\Delta\vec{L}$
$\vec{L}$
$\vec{\tau}$
$\vec{r}$
$\vec{\tau}$ points into the page.
$\vec{F}_g$
Gravity exerts a torque about the pivot; $\vec{\tau} = \vec{r} \times \vec{F}$ is into the page.


Direction of rotation
m
centripetal
force
F
$F = \frac{mv^2}{r}$

Procession



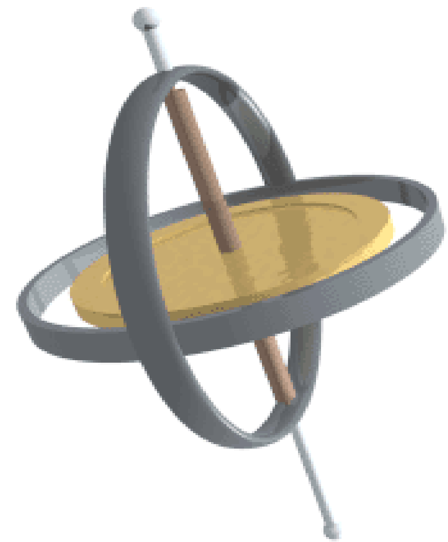
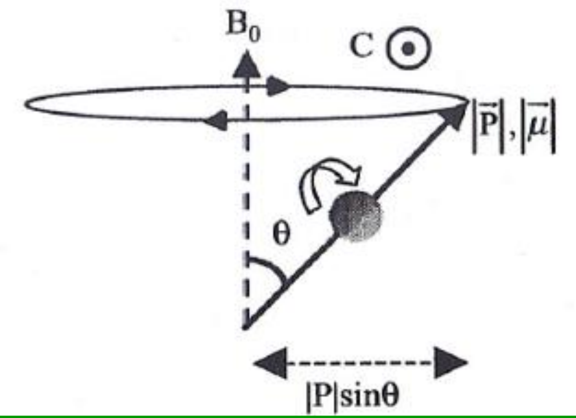


FIGURE 4.4. *(Left) Classically, the action of B_0 trying to align the proton magnetic moment along the direction of B_0 produces a torque C, the direction of which is out of the plane of the figure. (Right) An analogous situation of a spinning gyroscope precessing under the effect of gravity.*

Processional Frequency

$$\mathbf{C} = \boldsymbol{\mu} \times \mathbf{B}_0 = i_N \, |\mu| \, |B_0| \sin\theta \tag{4.13}$$

where i_N is a unit vector normal to both $\boldsymbol{\mu}$ and $\mathbf{B}_0$. The direction of the torque, shown



$$\mathbf{C} = \frac{d\mathbf{P}}{dt} = \boldsymbol{\mu} \times \mathbf{B}_0 \tag{4.14}$$

Note: Due to Lorentz Force

$$\sin(d\phi) = \frac{d\mathbf{P}}{|\mathbf{P}| \sin\theta} = \frac{\mathbf{C}\, dt}{|\mathbf{P}| \sin\theta} \tag{4.15}$$

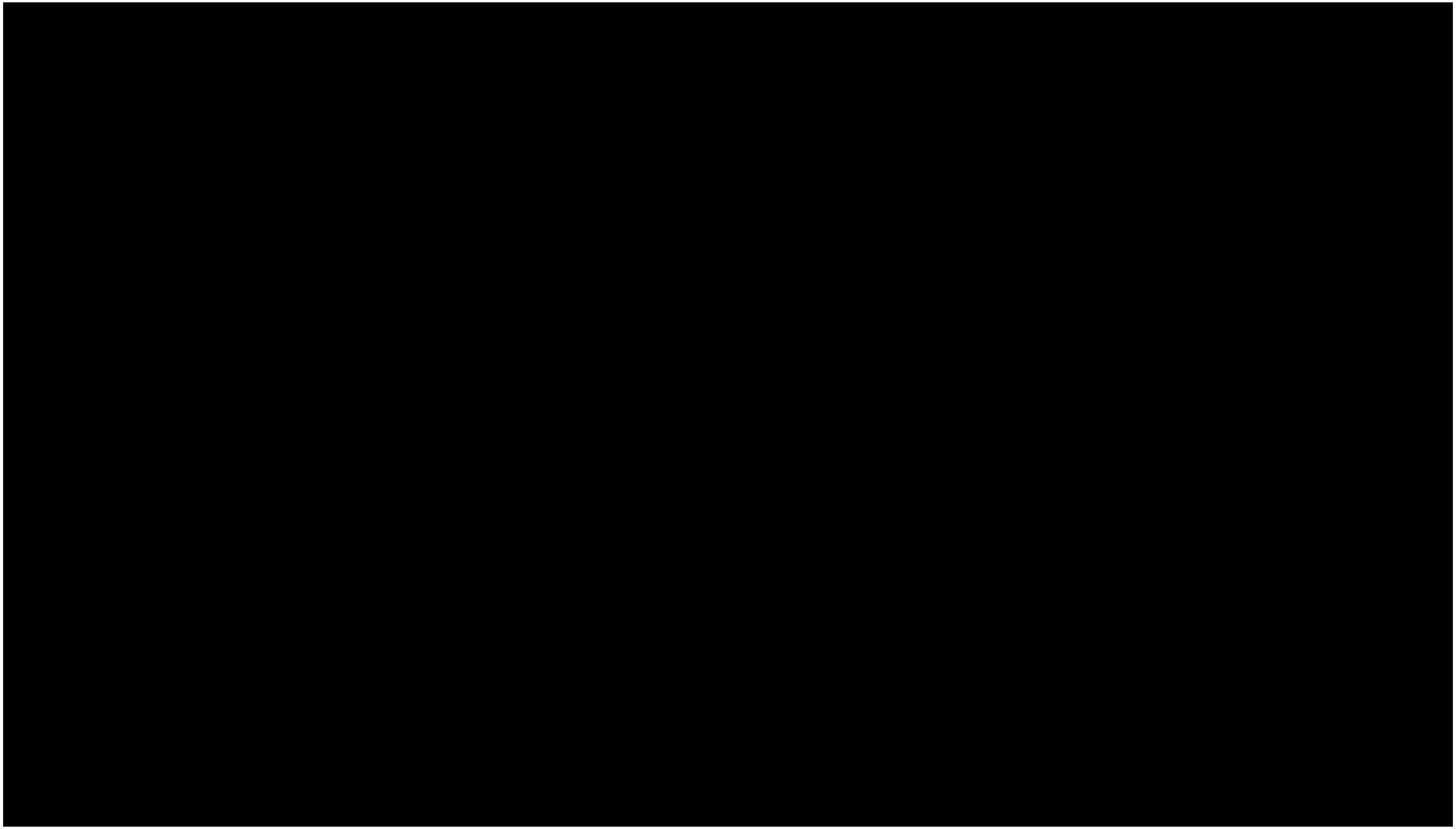
If $d\phi$ is small, then $\sin(d\phi) \sim d\phi$. The angular precessional frequency ω is given by $d\phi/dt$ and so can be evaluated as

$$\omega = \frac{d\phi}{dt} = \frac{\mathbf{C}}{|\mathbf{P}| \sin\theta} = \frac{\boldsymbol{\mu} \times \mathbf{B}_0}{|\mathbf{P}| \sin\theta} = \frac{\gamma \mathbf{P} \times \mathbf{B}_0}{|\mathbf{P}| \sin\theta} \tag{4.16}$$

Expanding the cross product gives

$$\omega = \frac{\gamma \, |\mathbf{P}| \, |\mathbf{B}_0| \sin\theta}{|\mathbf{P}| \sin\theta} = \gamma B_0 \tag{4.17}$$

Nice Precession Demo!



Magnetization

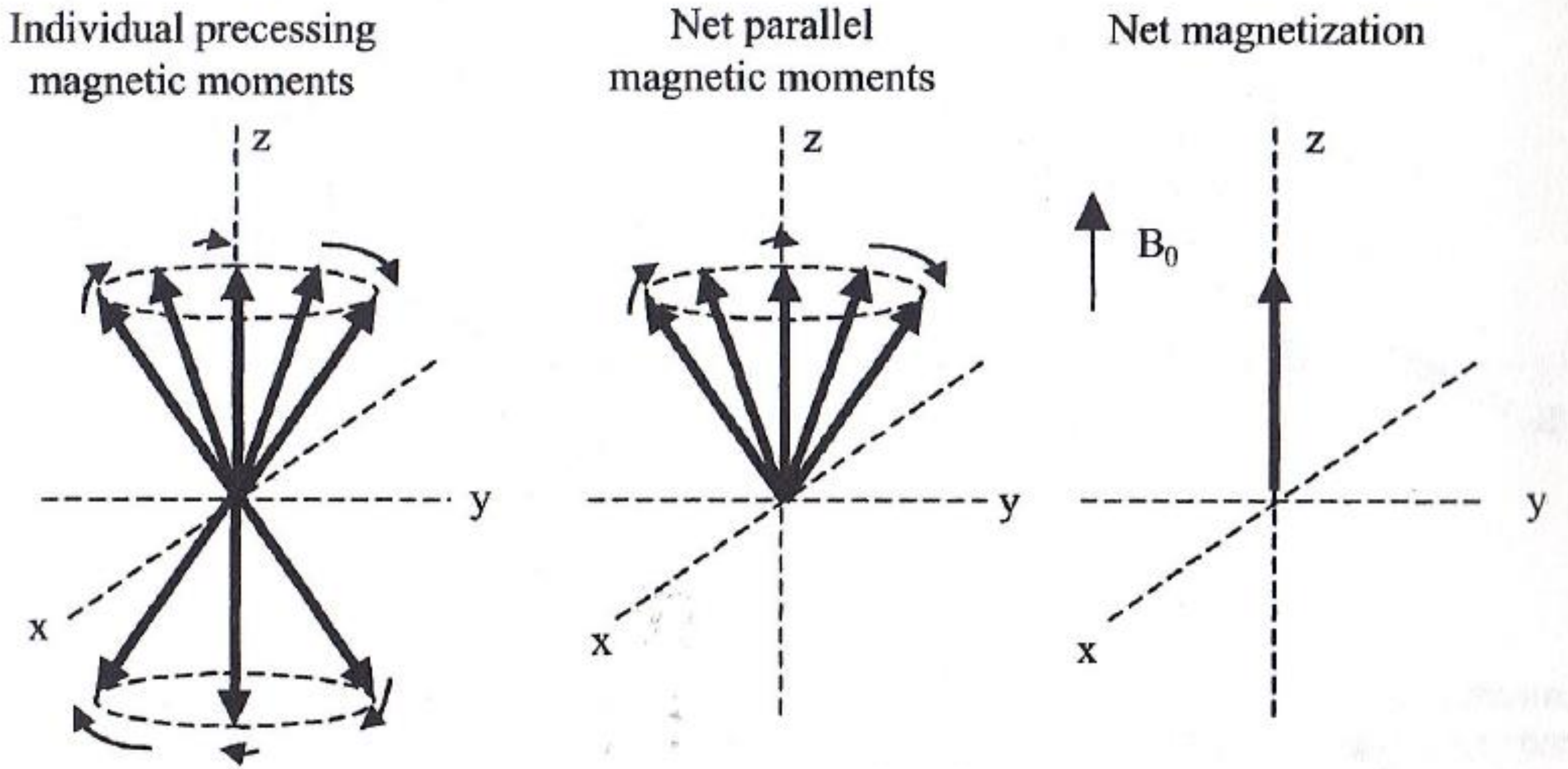


FIGURE 4.5. *(Left) The magnetic moments of each proton precess around the z axis with a frequency $\omega = \gamma B_0$ and at an angle of 54.7° to this axis. (Center) The magnetic moments corresponding to the slightly greater number aligned parallel to the magnetic field than antiparallel. (Right) The vector sum of all of the magnetic moments only has a z (longitudinal) component, with no component in the xy (transverse) plane.*

Steady State

$$M_0 = \sum_{n=1}^{N_s} \mu_{z,n} = \frac{\gamma h}{4\pi}\left(N_{\text{parallel}} - N_{\text{antiparallel}}\right) = \frac{\gamma^2 h^2 B_0 N_s}{16\pi^2 kT} \tag{4.20}$$

When the patient is placed in the magnetic field, this net magnetization can be considered as the vector sum of all of the individual proton magnetic moments. These magnetic moments precess around B_0, and are randomly distributed around a "precession cone" as shown in Figure 4.5. The net magnetization only has a z component because the vector sum of the components in the x and y axes is zero:

$$M_z = M_0, \qquad M_y = 0, \qquad M_x = 0 \tag{4.21}$$

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Motivation for MR Signals

M_0 Reflects the Water/Lipid but How to Measure M_0?

Electromagnetic Induction Seems the Way to Go but M_0 Itself Does not Give a Changing Field.

How to Let M0 Make an Alternating Field?

Flip M_0 to Have a Transverse Component Which Will Produce an Alternating Magnetic Field Due to Precession

1946: Nuclear Magnetic Resonance (NMR)
Bloch (Stanford) and Purcell (Harvard)

1952 Nobel Prize in Physics

How to Perturb the Balance?

$$\Delta E = \frac{\gamma h \mathbf{B}_0}{2\pi} \tag{4.9}$$

resonance frequency. The frequency f of this electromagnetic field can be calculated from

$$hf = \Delta E = \frac{\gamma h B_0}{2\pi} \tag{4.18}$$

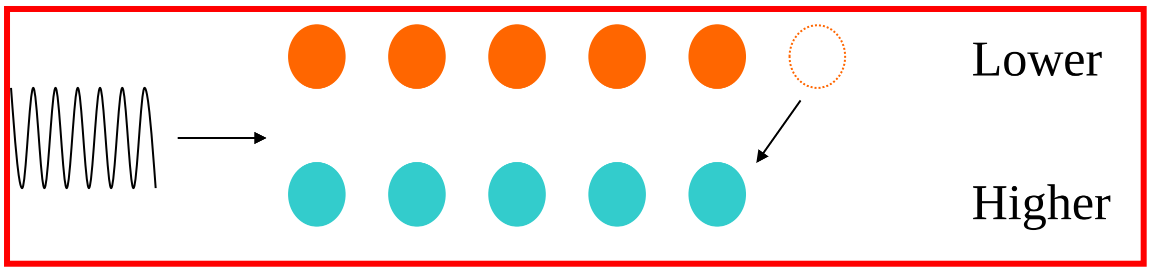
resulting in equations for the resonant frequency in hertz (f) or radians per second (ω):

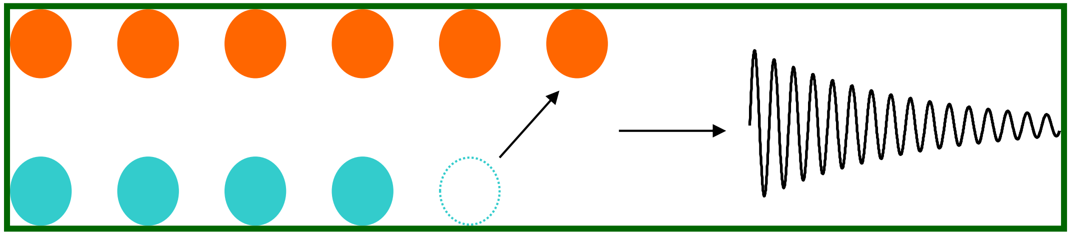
$$f = \frac{\gamma B_0}{2\pi}, \qquad \omega = \gamma B_0 \tag{4.19}$$

The value of f for a 1.5-T clinical scanner is approximately 63.9 MHz. By comparing equations (4.17) and (4.19) it can be seen that the Larmor precession frequency is identical to the frequency of the electromagnetic field that must be applied for transitions to occur between the parallel and the antiparallel energy levels in the quantum mechanical model.

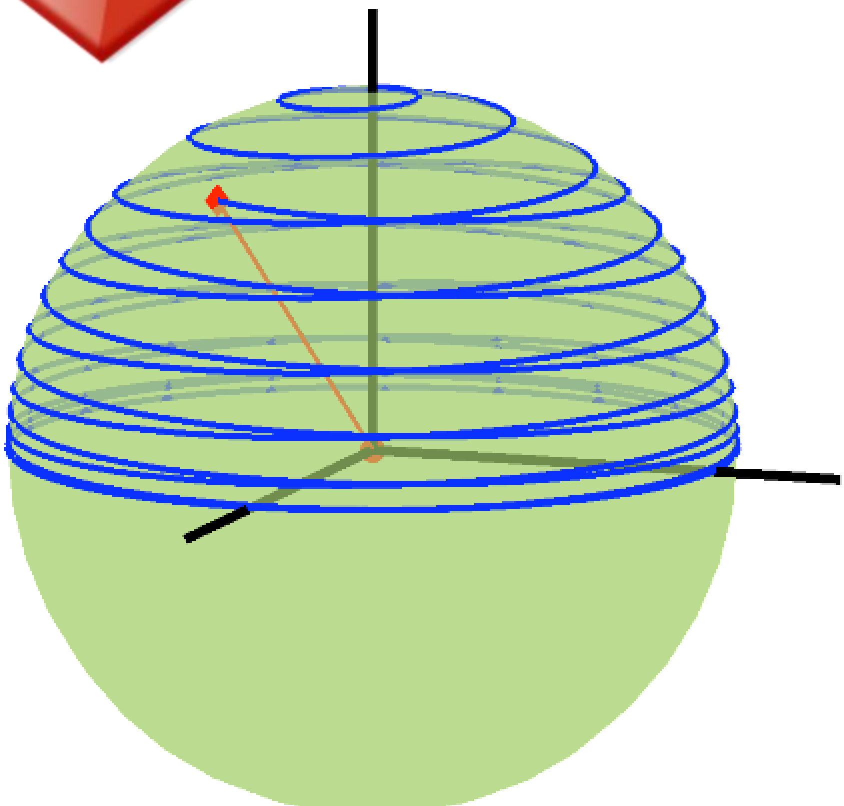
RF Excitation (QM View)

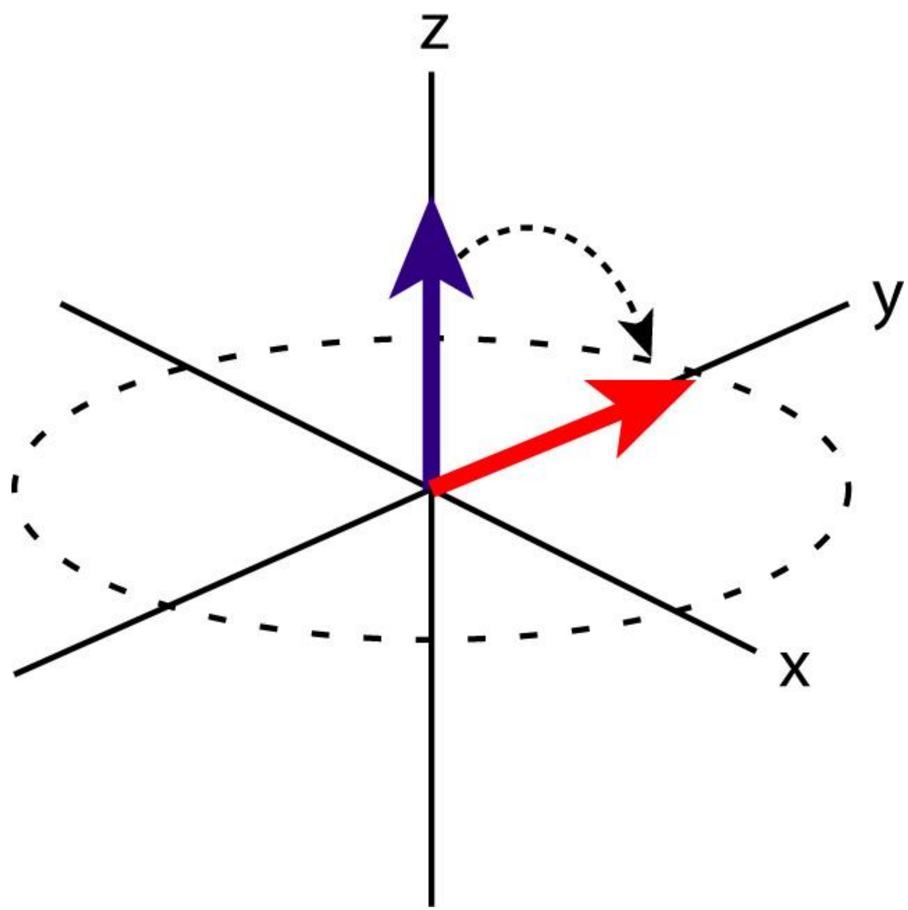
- RF to Manipulate the Magnetization
- RF Only Effective at the Resonance Frequency
- Perturbed Nuclei to Generate the Same RF Signal
- Emitted Signals to Be Detected Externally





Rotating Field (Classic View)





Lab & Rotating Frames

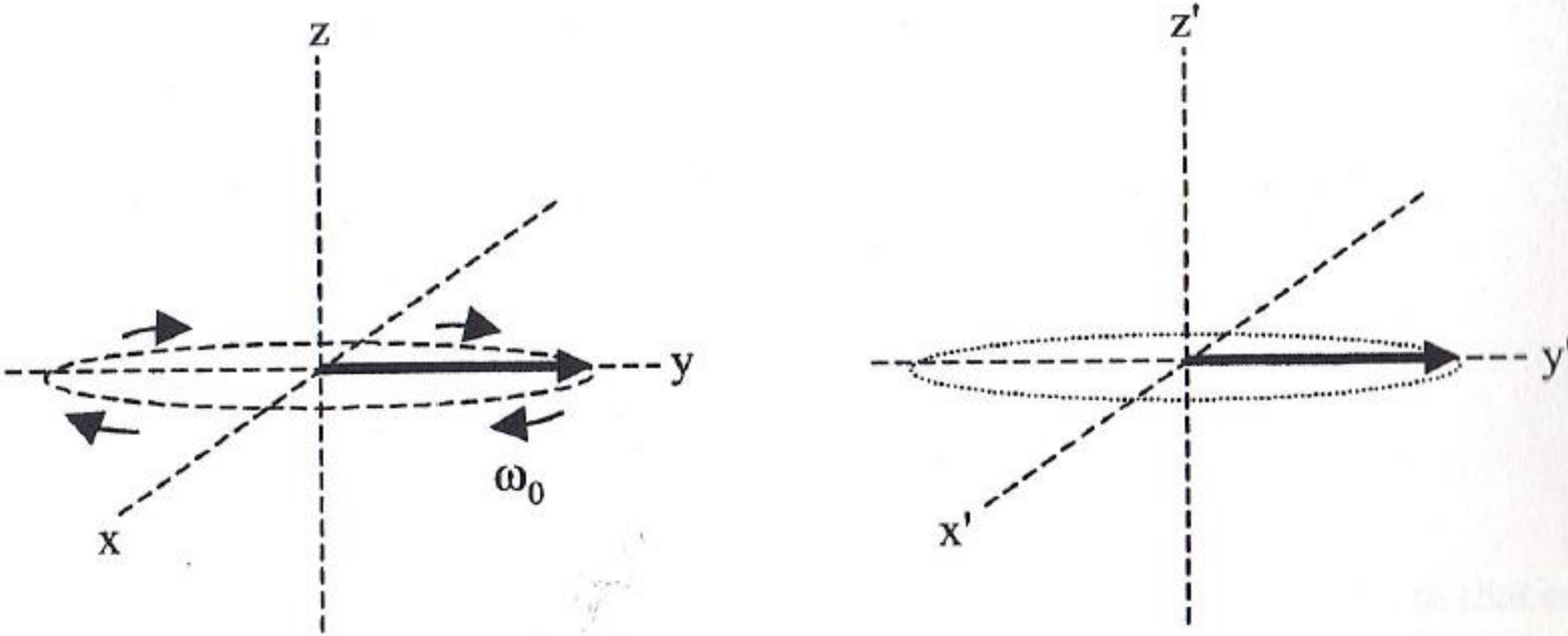


FIGURE 4.7. *An illustration of the evolution of magnetization in the rotating reference frame. (Left) In the "laboratory frame," the net magnetization precesses around the z axis at the Larmor frequency. (Right) In the rotating reference frame, the x′y′ plane rotates around the z′ axis at the Larmor frequency, and therefore the net magnetization is static.*

Bloch Equation

Bloch equations

In 1946 Felix Bloch formulated a set of equations that describe the behavior of a nuclear spin in a magnetic field under the influence of *rf* pulses. He modified Equation 13 to account for the observation that nuclear spins "relax" to equilibrium values following the application of *rf* pulses. Bloch assumed they relax along the z-axis and in the x-y plane at different rates but following first-order kinetics. These rates are designated $1/T_1$ and $1/T_2$ for the z-axis and x-y plane, respectively. T_1 is called spin-latice relaxation and T_2 is called spin-spin relaxation. Both of these will be described in more detail later in the class. With the addition of relaxation, Equation 13 becomes

$$\frac{d\mathbf{M}(t)}{dt} = \mathbf{M}(t) \times \gamma \mathbf{B}(t) - \mathbf{R}(\mathbf{M}(t) - M_0) . \qquad (1\text{-}14)$$

where **R** is the "relaxation matrix". Equation 14 is best understood by considering each of its components:

$$\frac{dM_z(t)}{dt} = \gamma[M_x(t)B_y(t) - M_y(t)B_x(t)] - \frac{M_z(t) - M_0}{\mathrm{T}_1}$$

$$\frac{dM_x(t)}{dt} = \gamma[M_y(t)B_z(t) - M_z(t)B_y(t)] - \frac{M(t)_x}{\mathrm{T}_2}$$

$$\frac{dM_y(t)}{dt} = \gamma[M_z(t)B_x(t) - M_x(t)B_z(t)] - \frac{M(t)_y}{\mathrm{T}_2}$$

Class Notes for BCH 6746 (Spring, 2000)
Structural Biology: Theory and Applications of NMR Spectroscopy

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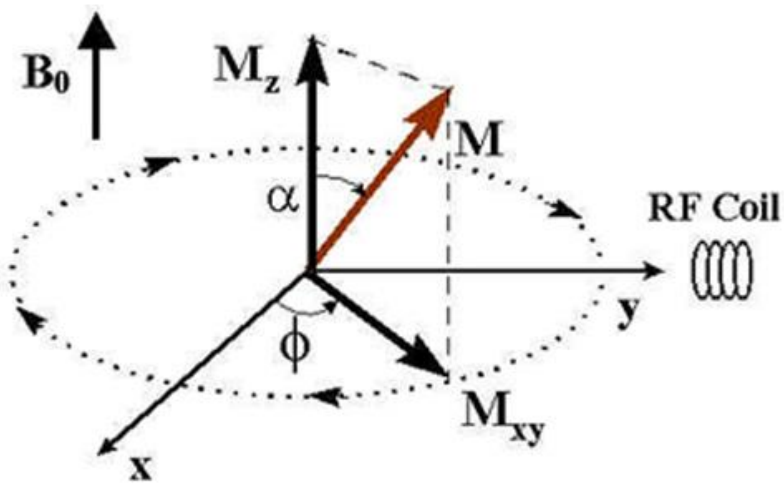
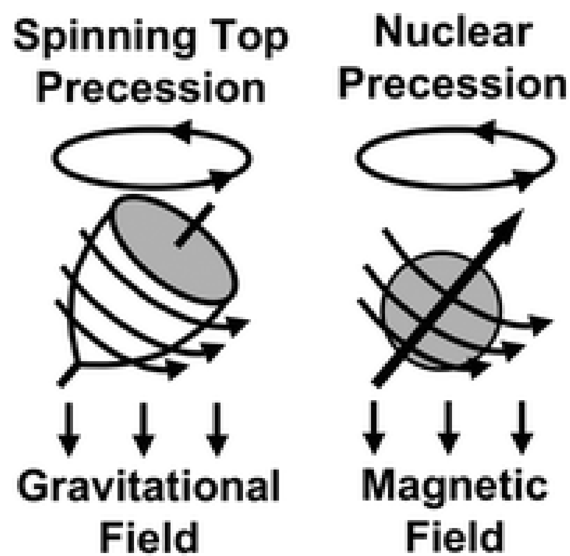
Bloch Equation in Working

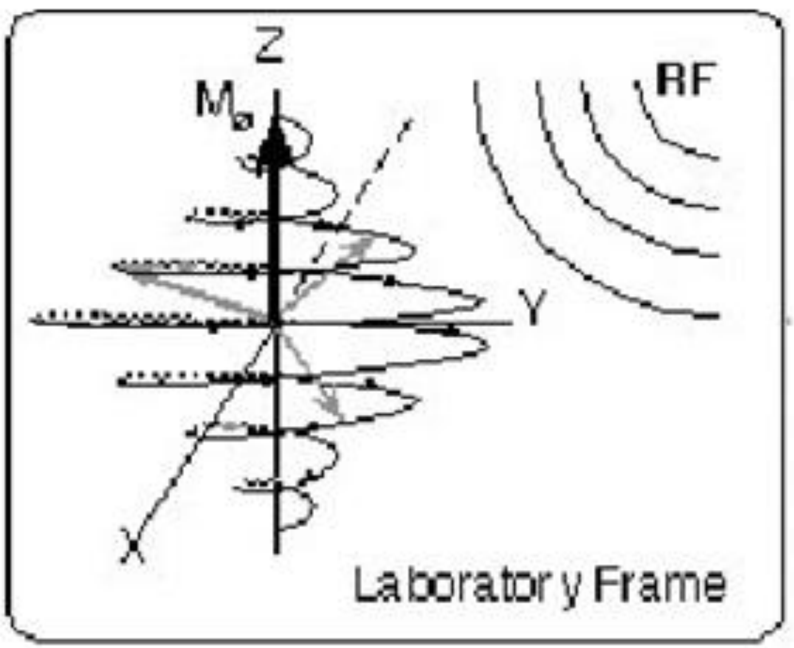
- Precession
  - If M Not Parallel to B, Then M in Precession
  - If M Parallel to B, Then No (Zero) Precession
- Flip M
  - If M Not Parallel to Be, Then M_{xy} in Oscillation
  - M_z->M_0 (T1 Effect), M_{xy}->0 (T2 Effect)

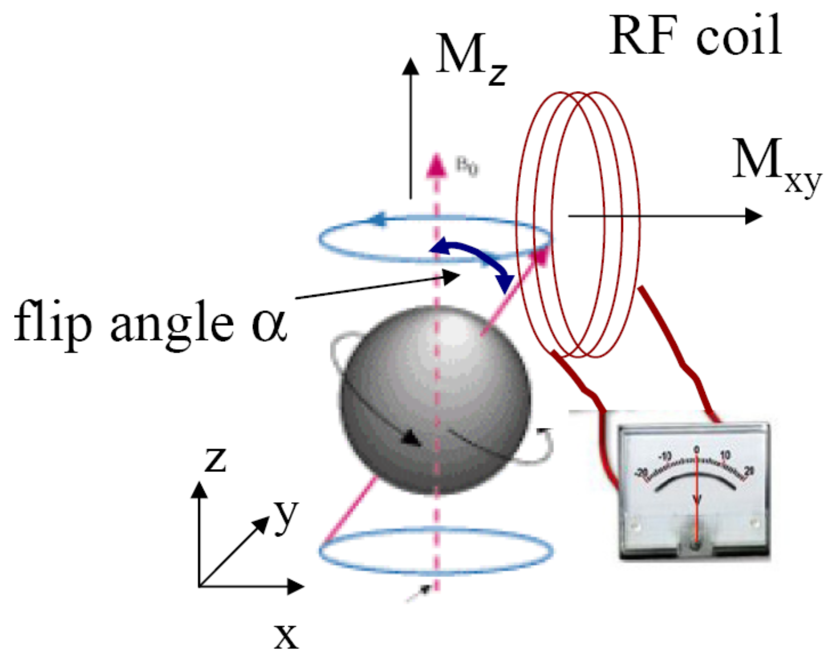




Signal Detection

- When M Flipped
  - M_{xy} Oscillates to induce a Sinusoidal Signal via Induction in an RF Coil at the Resonance Frequency
  - The Highest Induction Reached at the Flip Angle 90^o so that $M_{xy}=M_0$

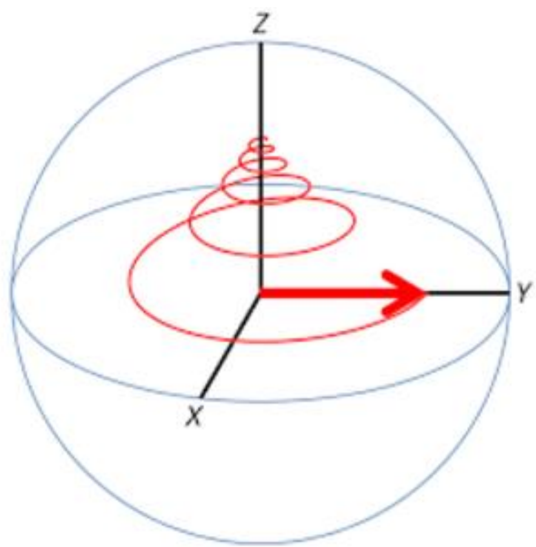


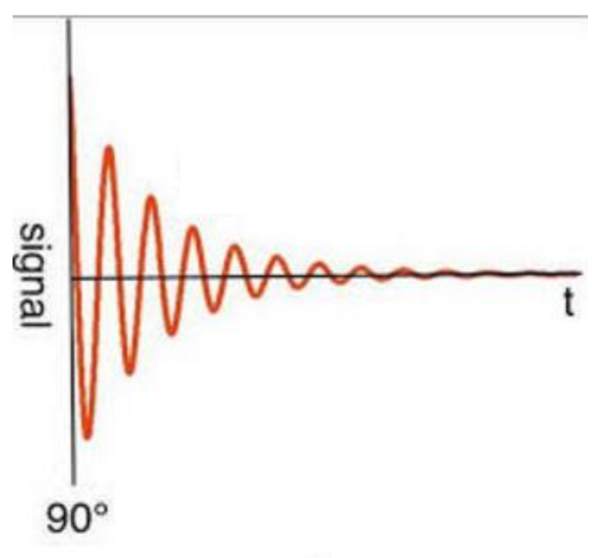


M Relaxation

When the RF Pulse is Off, M Will Re-align with B_0. The Excess Energy Will be Released. The RF Signal will Decay to Zero. This Free Induction Decay (FID) Gives

- **NMR Signal**
- **At the Resonance Frequency**
- **Proportional to Proton Density**





MRI Physics

- **Preview & Review**
- **Physical Foundation**
  - Magnetic Moment (Quantum Mechanics)
  - Magnetization (Classic Model)
  - Precession
- **Signal Generation**
  - RF Perturbation **(**Rotating Frame)
  - Bloch Equation (Parameters T1 & T2)
  - FID Detection
- **Signal Decay**
  - T1 & T2 Mechanism
  - Inversion Recovery & Spin Echo

Why Decay (FID)?

- **T1 Relaxation – Flipped Nuclei Realign with B0**
- **T2 relaxation – Flipped Nuclei Out of Phase**
- **T2* relaxation – Disturbance in the Magnetic Field (Susceptibility)**

The NMR Signal Proportional to the Proton Density, Reduced by T1, T2, and T2* Factors

Tissues Have Good MR Contrasts to Generate Images

- **T1 – Gray/White matter**
- **T2 – Tissue/Cerebrospinal Fluid Flow (CSF)**
- **T2* – Susceptibility (Functional MRI)**

T1: Returning to Steady State

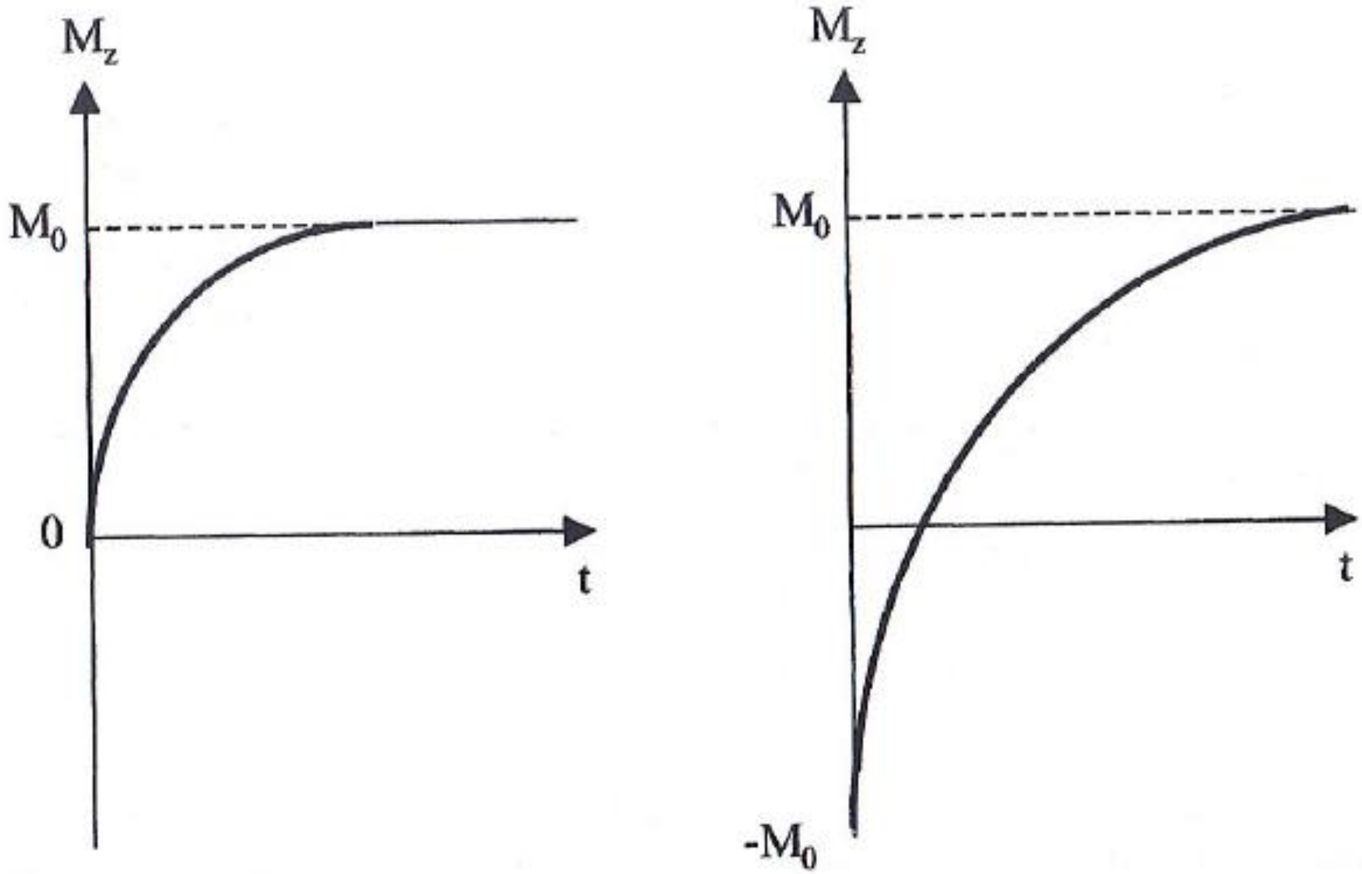


FIGURE 4.8. Plots of M_z versus time after (left) a 90° pulse and (right) a 180° pulse.

T2: De-phasing of Moments

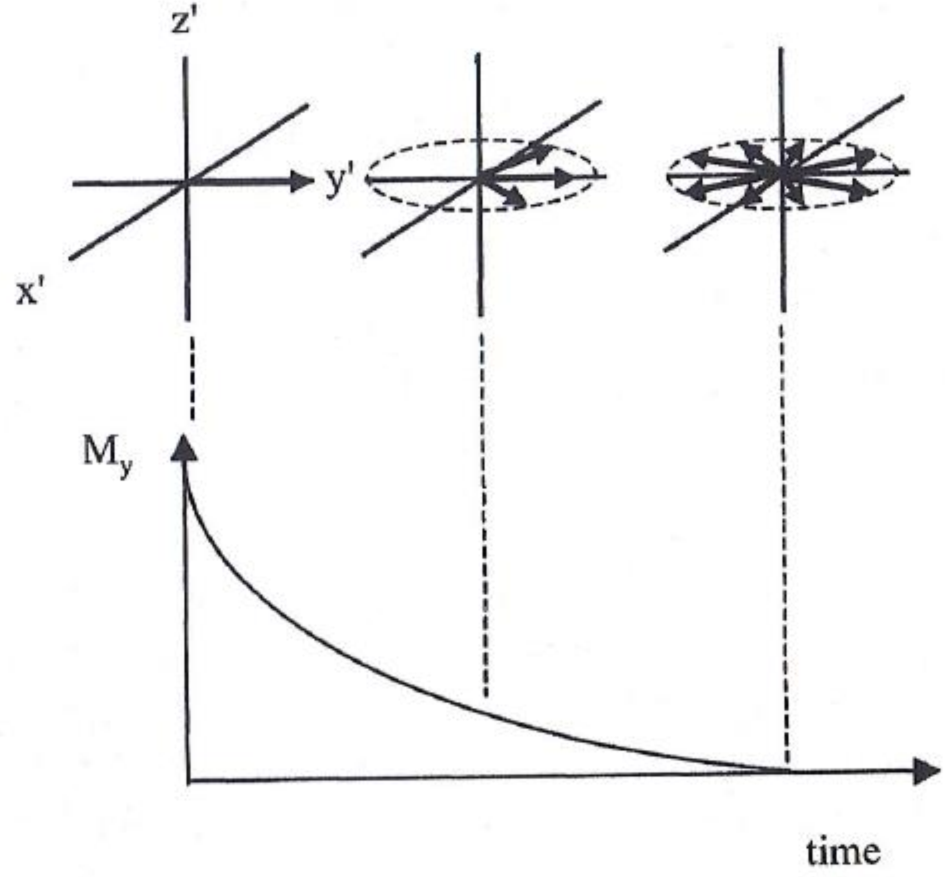


FIGURE 4.9. (Top) After a 90° pulse, the individual magnetic moments precess at different frequencies because they experience slightly different magnetic fields. (Bottom) The M_y component of magnetization decreases over time, and when the individual vectors are randomly distributed in the transverse plane, there is no net magnetic moment, and no signal is detected.

T1 & T2

component is given by $M_0 \cos\alpha$. The value of M_z at a time t after the RF pulse is given by

$$M_z(t) = M_0 \cos\alpha + (M_0 - M_0 \cos\alpha)\left(1 - e^{-t/T_1}\right) \tag{4.25}$$

For example, after a 90° pulse the value of M_z is given by

$$M_z(t) = M_0\left(1 - e^{-t/T_1}\right) \tag{4.26}$$

The M_x and M_y components of magnetization relax back to their thermal equilibrium values of zero with a time constant termed the spin–spin (T_2) relaxation time:

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2}, \qquad \frac{dM_y}{dt} = -\frac{M_y}{T_2} \tag{4.27}$$

T1 & T2 at 1.5T

TABLE 4.2. Tissue Relaxation Times at 1.5 T

Tissue	T_1 (ms)	T_2 (ms)
Fat	260	80
Muscle	870	45
Brain (gray matter)	900	100
Brain (white matter)	780	90
Liver	500	40
Cerebrospinal fluid	2400	160

T_2, T_2^+ & T_2^*

In fact, the loss in phase coherence of the transverse magnetization arises from two different mechanisms. The first is the "pure" T_2 decay outlined above. The second arises from spatial variations in the strength of the magnetic field within the body. There are two major sources for these variations. The first is the intrinsic magnet design, that is, it is impossible to design a magnet producing a perfectly uniform magnetic field over the entire patient. The second source is local variations in magnetic field due to the different magnetic susceptibilities of different tissues: this effect is particularly pronounced at air/tissue and bone/tissue boundaries. Together, these factors produce loss of phase coherence, which is characterized by a relaxation time T_2^+. The overall relaxation time that governs the decay of transverse magnetization is a combination of signal loss due to T_2 and T_2^+ effects, and is designated by T_2^*, the value of which is given by

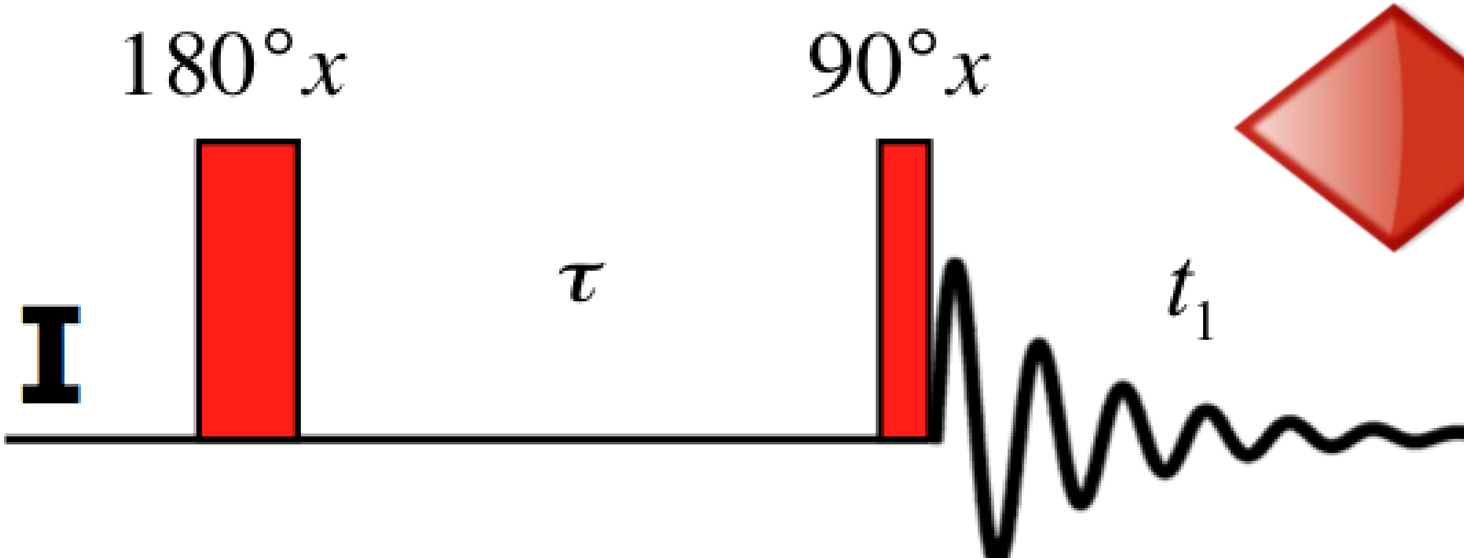
$$\frac{1}{T_2^*} = \frac{1}{T_2^+} + \frac{1}{T_2} \tag{4.29}$$

T1: Inversion Recovery

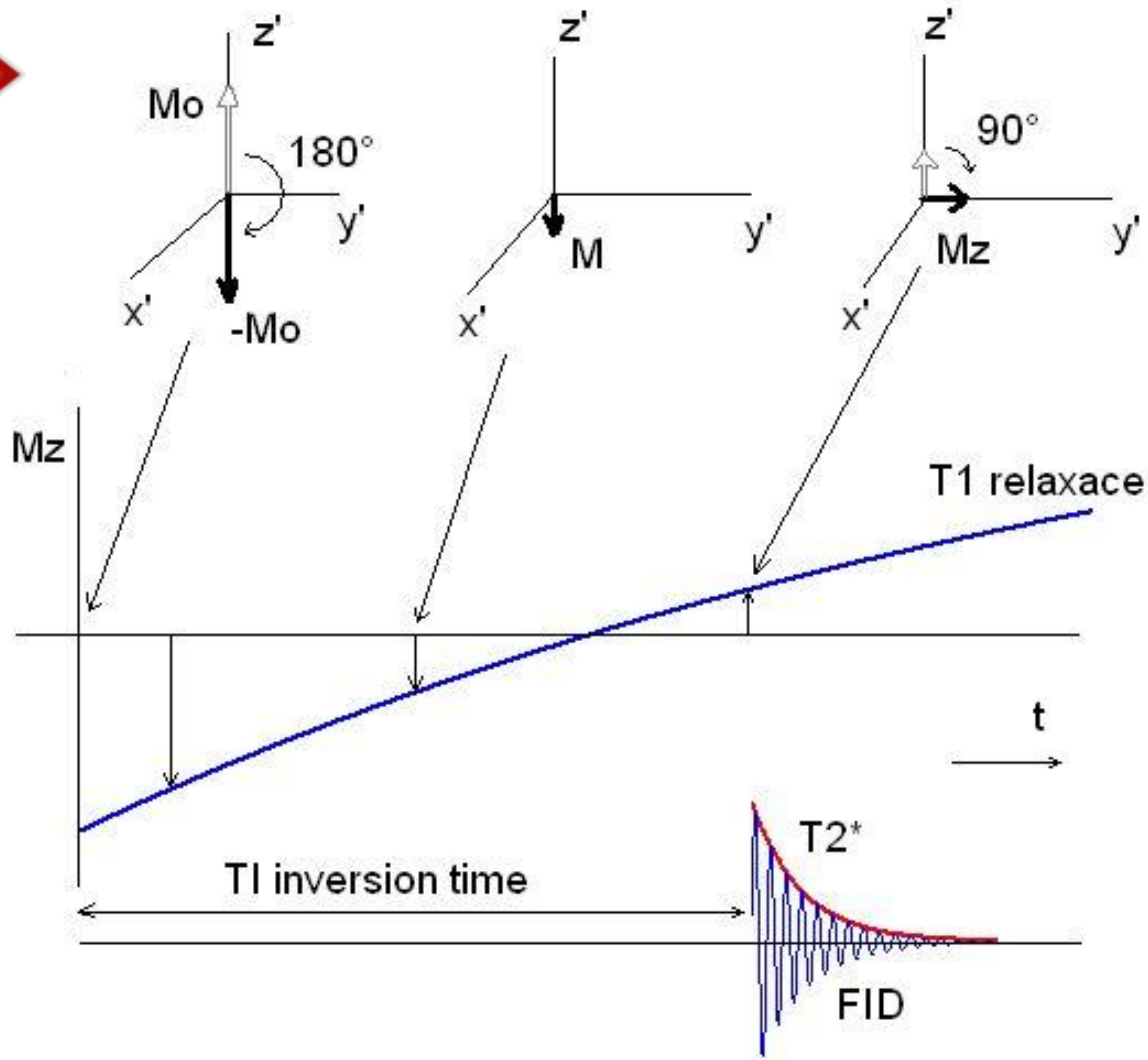
The value of T_1 is measured using an inversion recovery sequence, which consists of a 180° pulse, a variable delay τ, and a 90° pulse followed immediately by data acquisition. This sequence is repeated n times, each time with a different value of the variable delay. From equation (4.25) the detected signal $S(\tau_n)$ is given by

$$S(\tau_n) = M_0(1 - 2e^{-\tau_n/T_1}) \qquad (4.30)$$

A plot of ln $[S(\tau_n)]$ versus τ_n gives a straight line with a slope of $-T_1$.



Visualize it!



T2: Spin-Echo

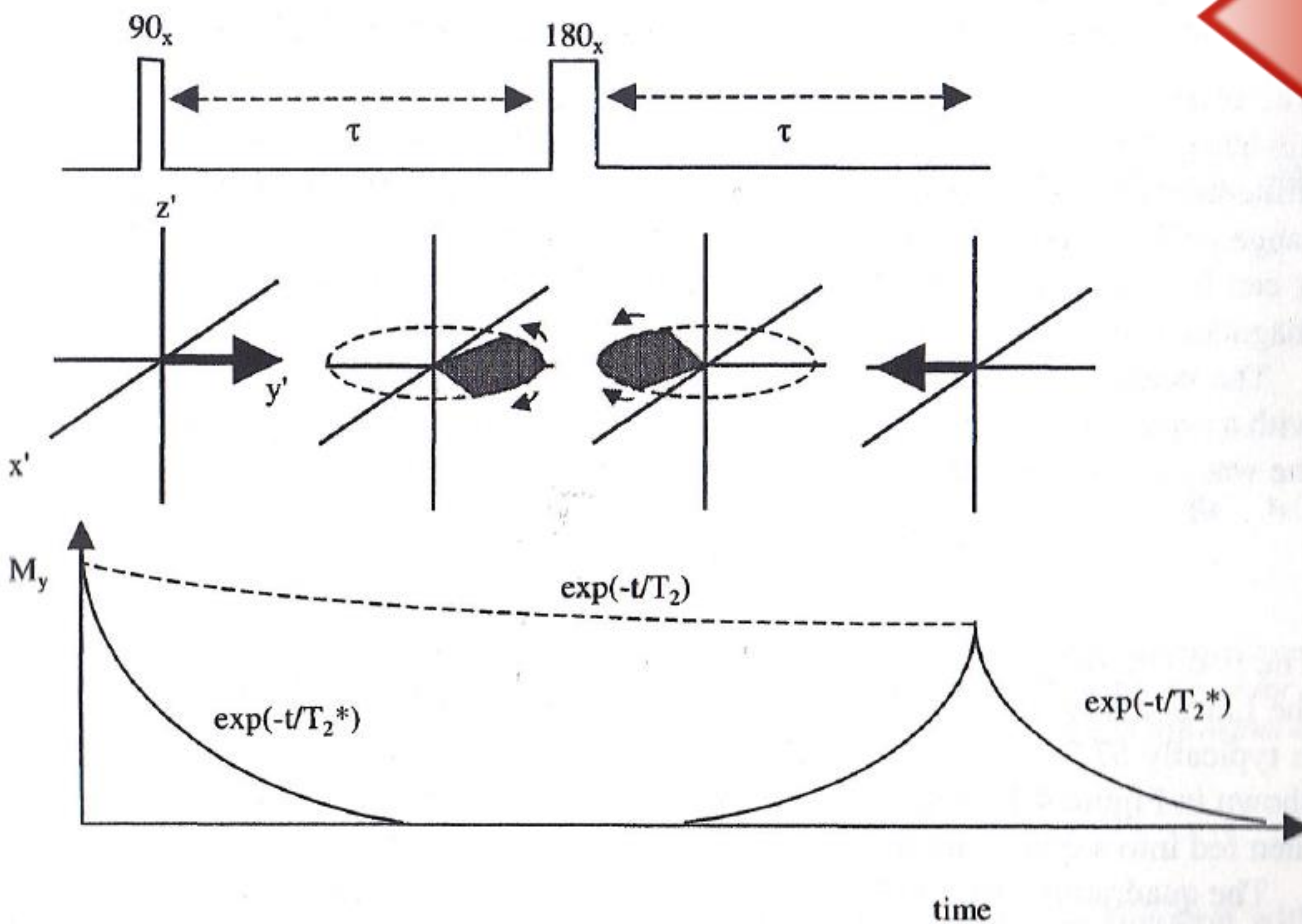


FIGURE 4.10. *A schematic of a spin-echo sequence. The 90° pulse tips the magnetization onto the y axis, where it decays with a time constant T_2^*. The effect of the 180° pulse is to "refocus" the magnetization such that at time τ after the 180° pulse, the individual vectors add constructively, and the signal reaches a peak.*

BB17 Homework

4.3. Calculate the effects of the following pulse sequences on thermal equilibrium magnetization. The final answer should include x, y, and z components of magnetization.

(a) 90_x (a pulse with tip angle 90°, applied about the x axis).

(b) 80_x.

(c) 90_x 90_y (the second 90° pulse is applied immediately after the first).

(d) 80_x 80_y.

4.4. Answer true or false with one or two sentences of explanation:

(a) Recovery of magnetization along the z axis after a 90° pulse does not necessarily result in loss of magnetization from the xy plane.

(b) A static magnetic field $\mathbf{B}_0$ that is homogeneous results in a free induction decay that persists for a long time.

(c) When the magnetization relaxes from the xy plane back to the z axis, it absorbs energy from the lattice.

(d) A short tissue T_1 indicates a slow spin–lattice relaxation process.

Due Date: Same (1 Working Week Later)