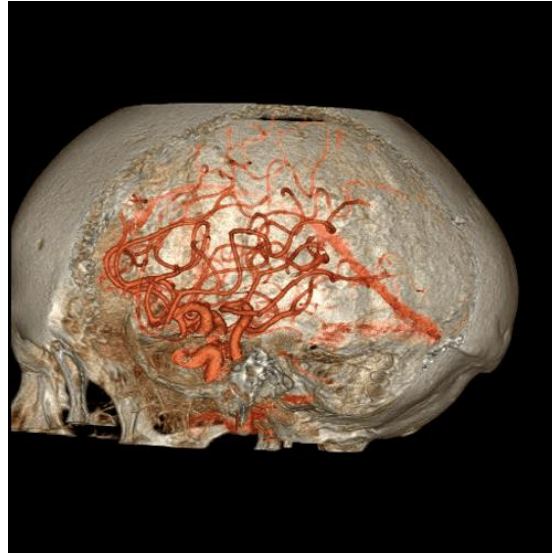


GE's "Revolution CT"



# MATLAB III: CT

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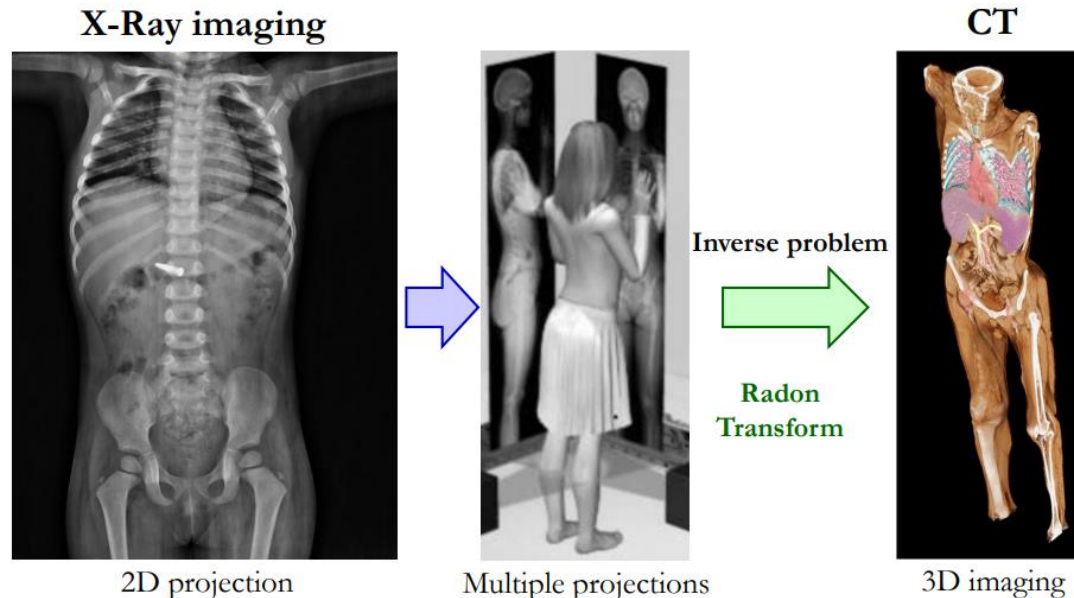
# Reminders

- Make sure you have MATLAB and the toolboxes (**Signal Processing, Image Processing**) installed
- Download the zip file from LMS and extract all files into your MATLAB folder

# Theory of CT

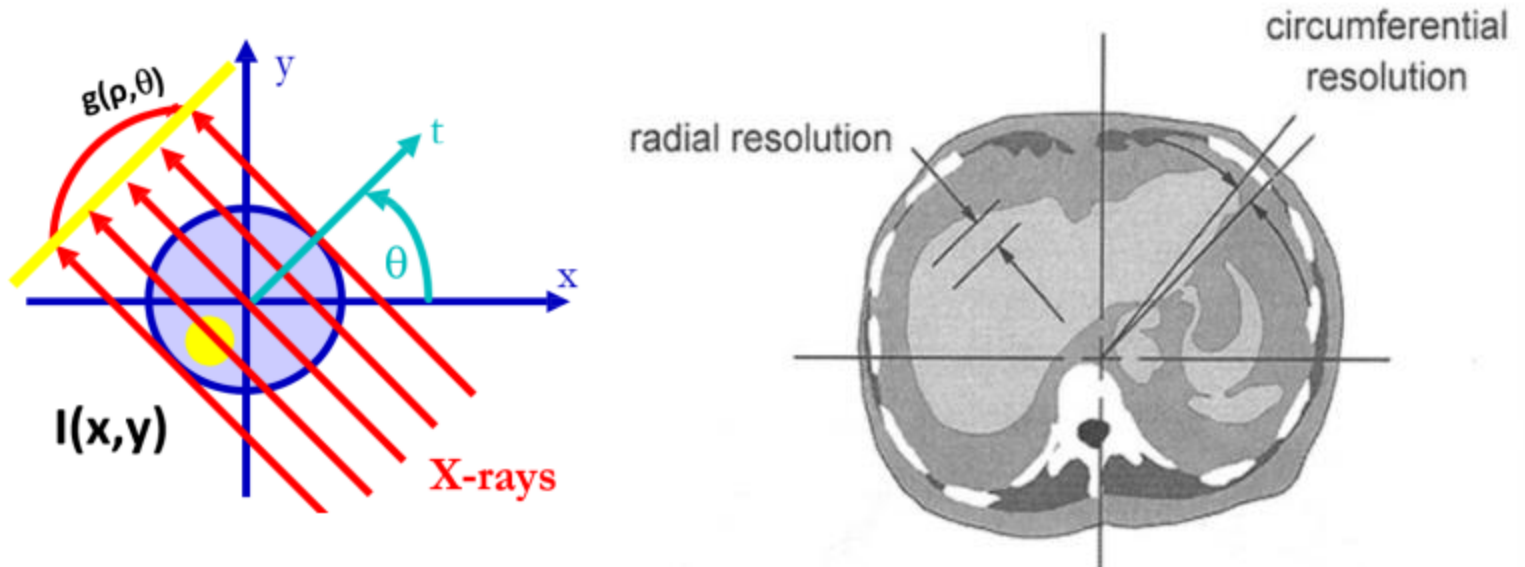
# General theory of CT

- X-ray projections are taken at multiple views
  - Attenuation due to different properties of tissue
- Solve the inverse problem to get tomographic information (CT = computed tomography)



# Resolution of CT reconstruction

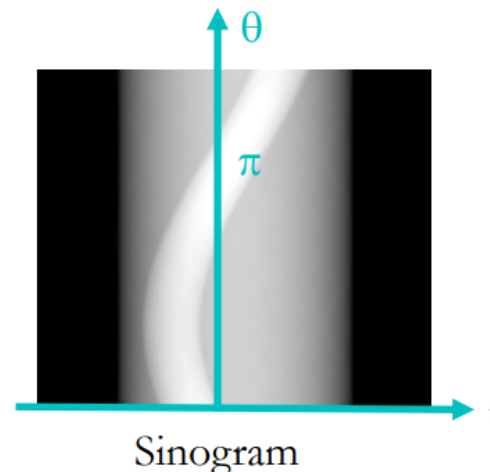
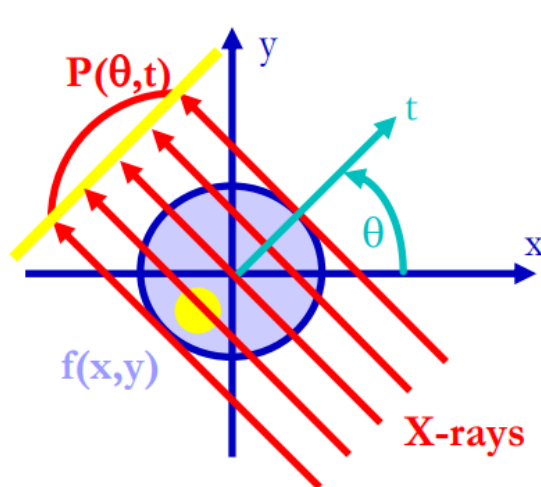
- **Number of rays** affects the radial component of spatial resolution
- **Number of views** affects the circumferential component of resolution



# Radon transform

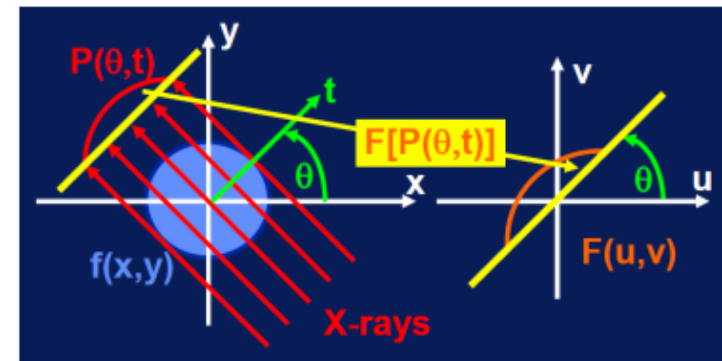
- Representation of the X-ray projection data in the form of a **sinogram**
- Indicates characteristics of the sample
  - Objects closer to the field of view produce a high amplitude in the sinogram

$$F(\theta, s) = \int_{-\infty}^{+\infty} f(s\cos\theta + t\sin\theta, -t\cos\theta + s\sin\theta) dt$$



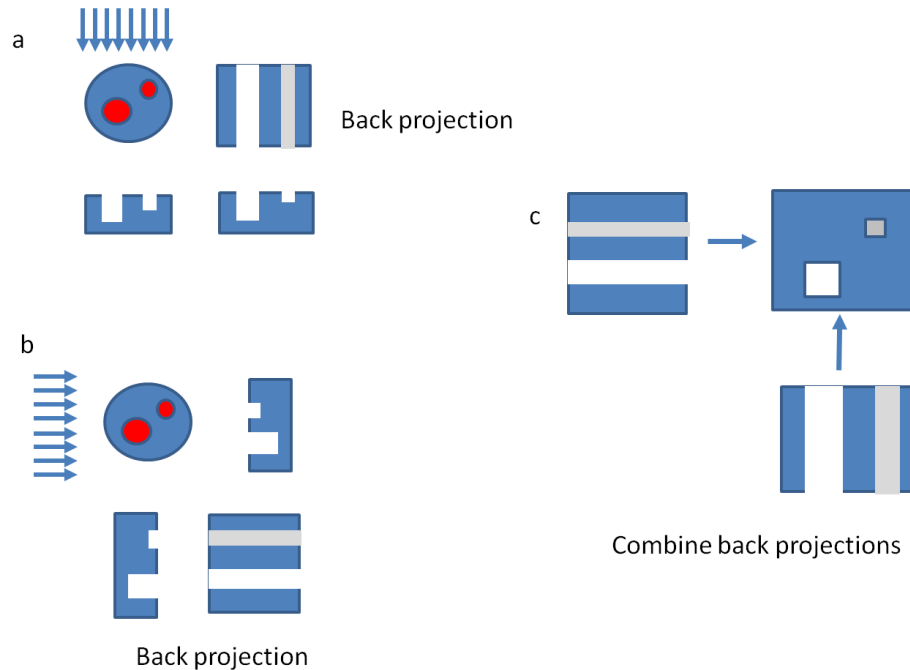
# Inverse radon transform: Fourier slice theorem

- 1D Fourier Transform (FT) of the RT projection profile acquired at angle  $\varphi$  is equivalent to the value of the 2D FT of  $f(x,y)$  along a line at the inclination angle  $\varphi$
- Putting together RT profiles at all acquisition angles yields the full 2D FT
- Image can be reconstructed using the inverse 2D FT



# Back-projection

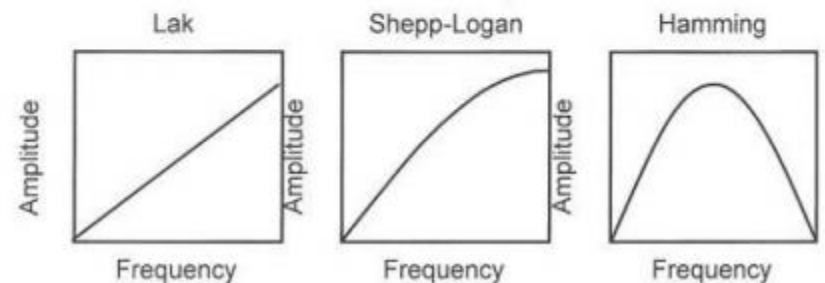
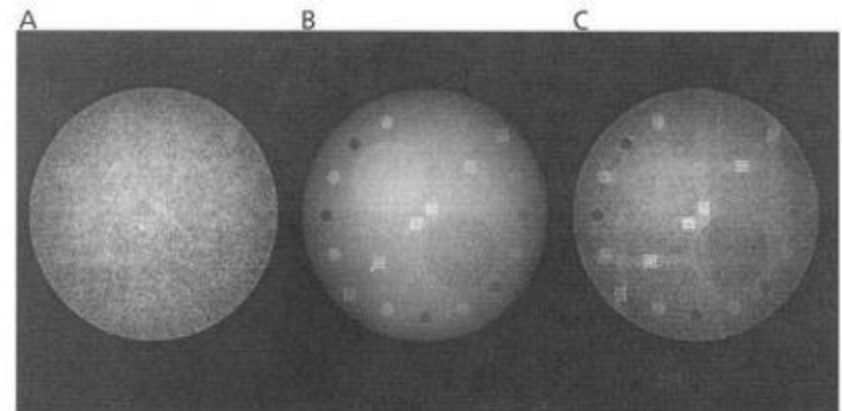
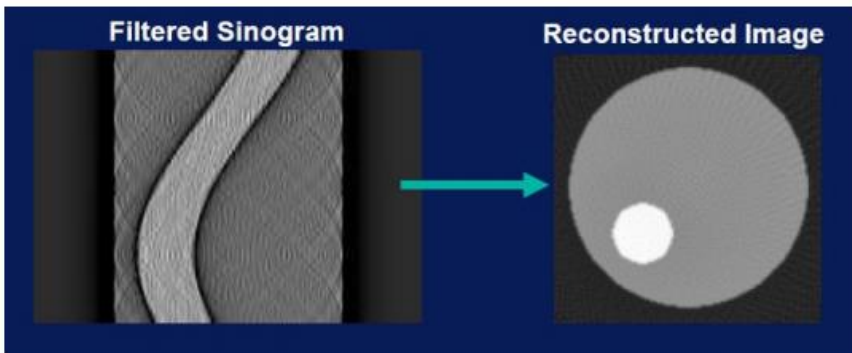
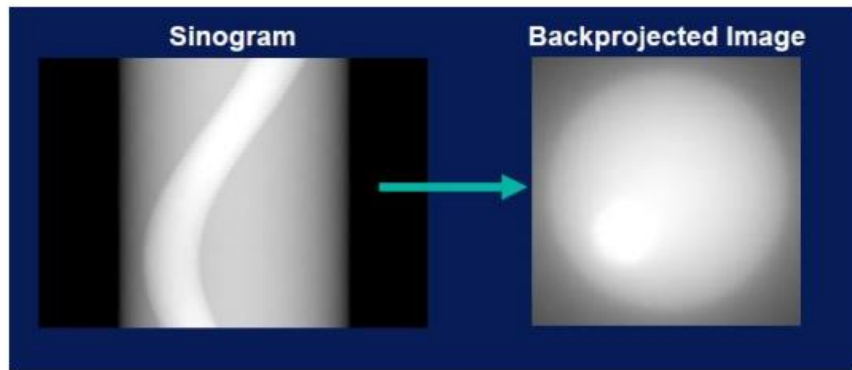
- Standard method of reconstructing CT slices
- Sinogram is used to back-project each view, then all views are combined to get the whole image



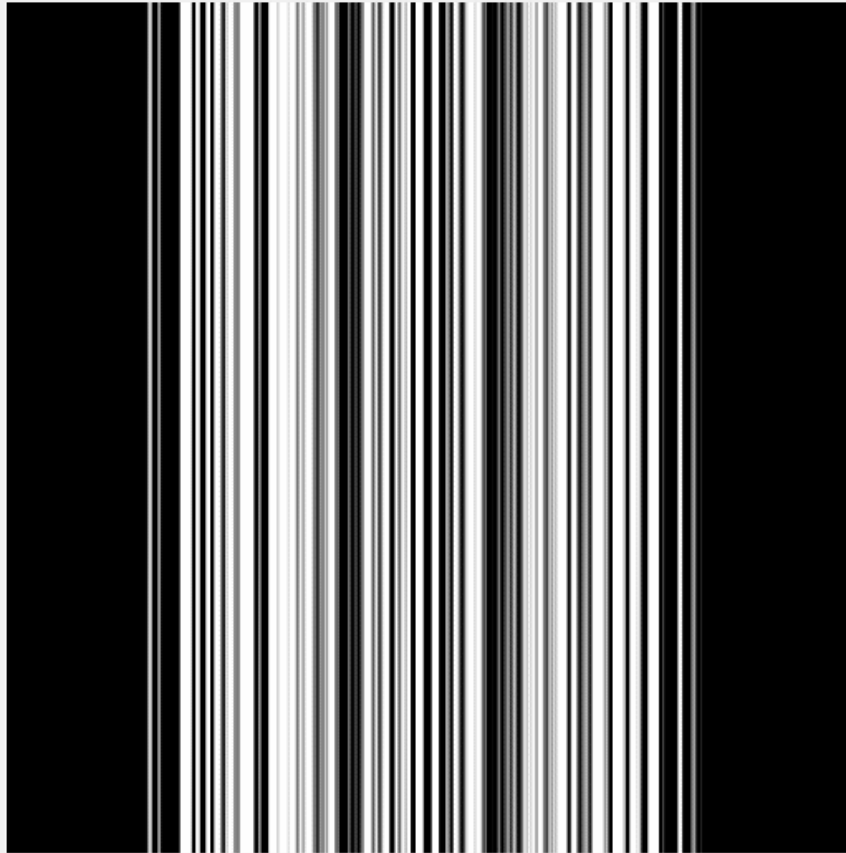


# Filtered back-projection (FBP)

- Unfiltered back-projection from a normal sinogram can produce a blurry image
  - A filter is required in the sinogram space



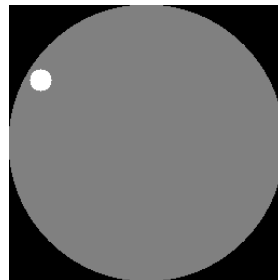
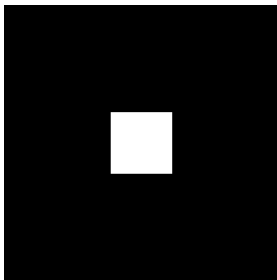
# FBP in progress



# How can we use MATLAB for CT?

1. Parallel beam CT

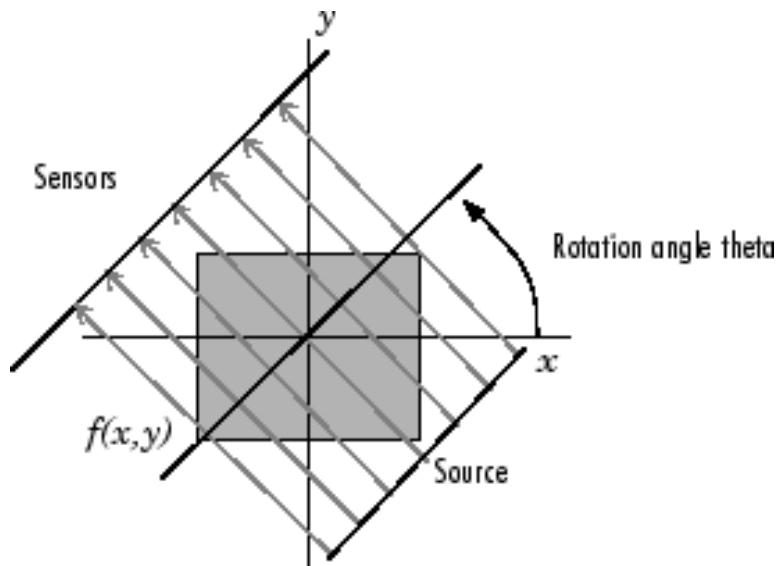
2. Fan beam CT



# Parallel beam CT

# Parallel beam: *radon*

- Beams (1 pixel apart) from the source are projected parallel to the detector and sample
- Parallel beams and detector are rotated around the center of the image at an angle theta  $\theta$



$[R, xp] = \text{radon}(I, \text{theta});$

**R** = value

**xp** = detector locations

**I** = input image

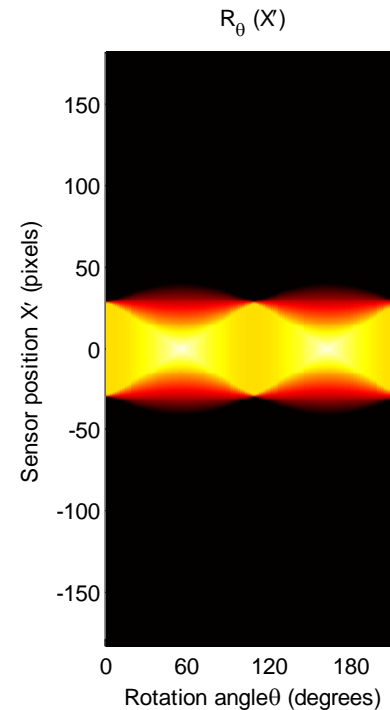
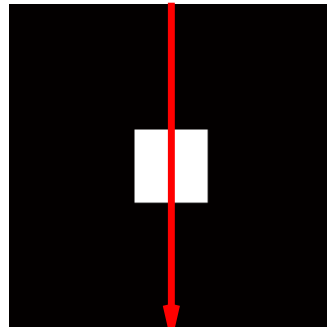
**theta** = rotation angle

\*Change p\_type (Line 5) to 1, 2, or 3 for different sample types\*

# Example 1: Parallel beam

- Run Ex1\_parallel.m
- As  $\theta$  (red arrow) is changing, information is added to the sinogram!

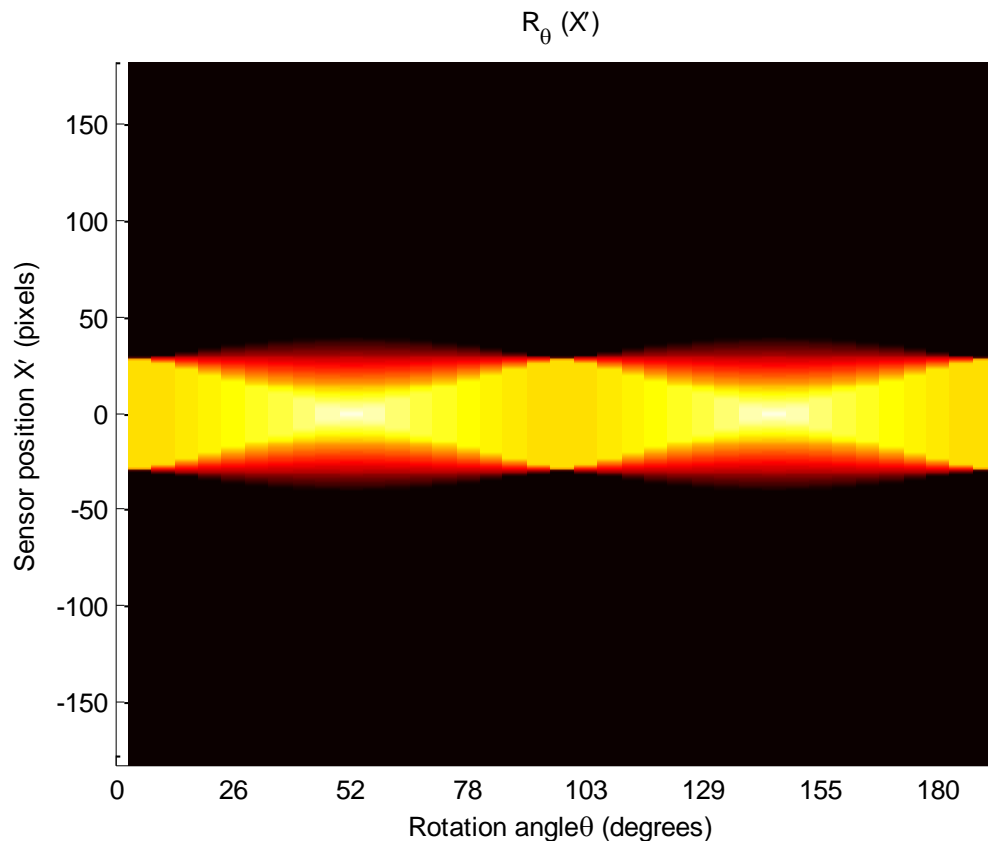
Image domain



Sinogram domain

# Example 1: Parallel beam

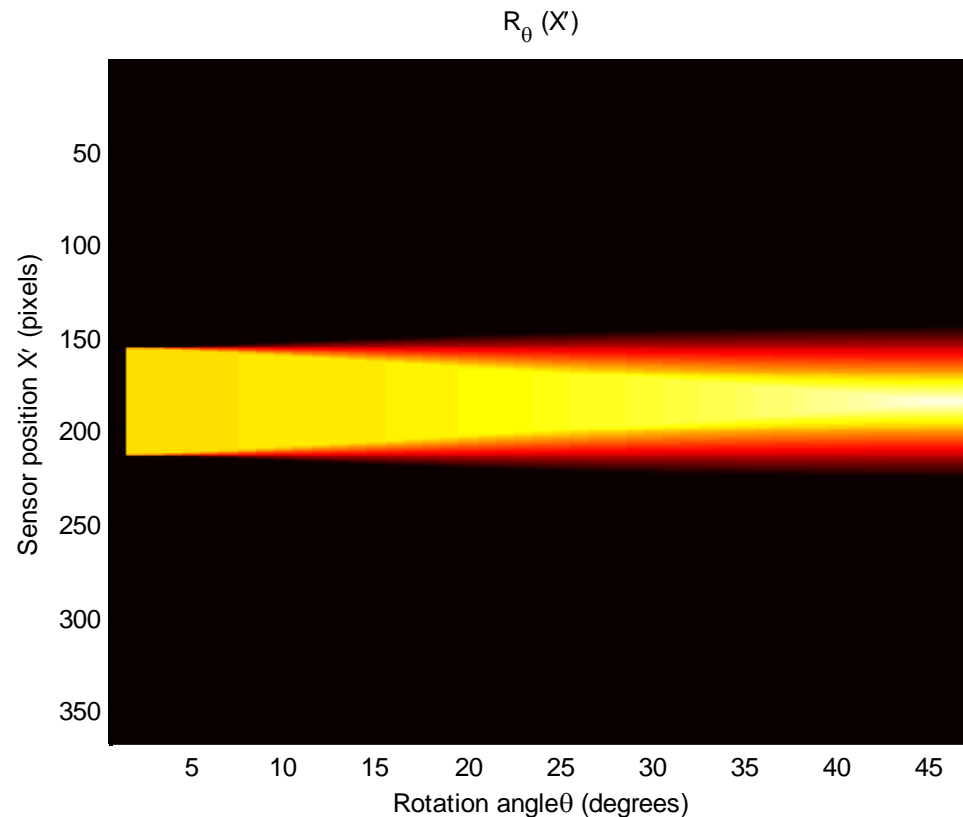
- What happens when we change the  $\theta$  step (**Line 23**) to 5? What is happening here?



\*Collected angles are more discrete (fewer angular views, lower resolution)\*

# Example 1: Parallel beam

- What happens if you change the max  $\theta$  (**Line 22**) to 45? What is happening here?



\*The sinogram is truncated (only part of sample is scanned)\*



# Parallel beam back-projection: *iradon*

- Back-projection for parallel beam sinograms
- Filtered back-projection can be implemented

$I = \text{iradon}(R, \text{theta}, \textit{interp}, \textit{filter});$

**I** = reconstructed image

**R** = sinogram

**theta** = rotation angle

***interp*** = interpolation method

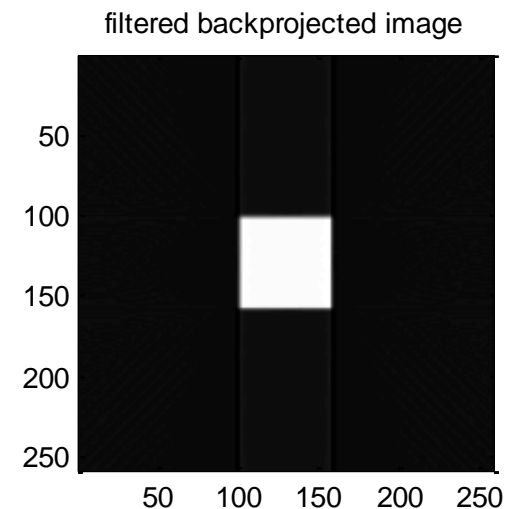
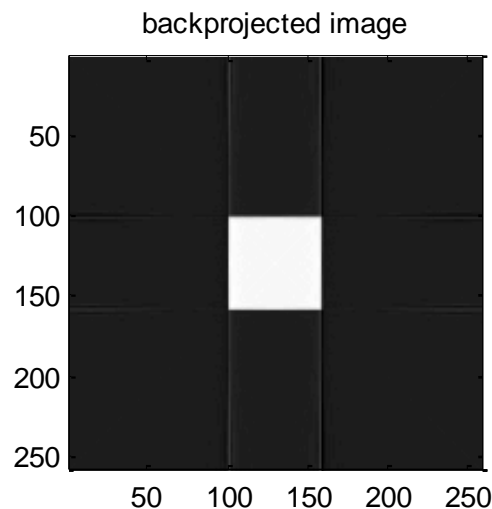
***filter*** = filter to be used for FBP

(*interp*, *filter* are optional)

\*Change p\_type (Line 5) to 1, 2, or 3 for different sample types\*

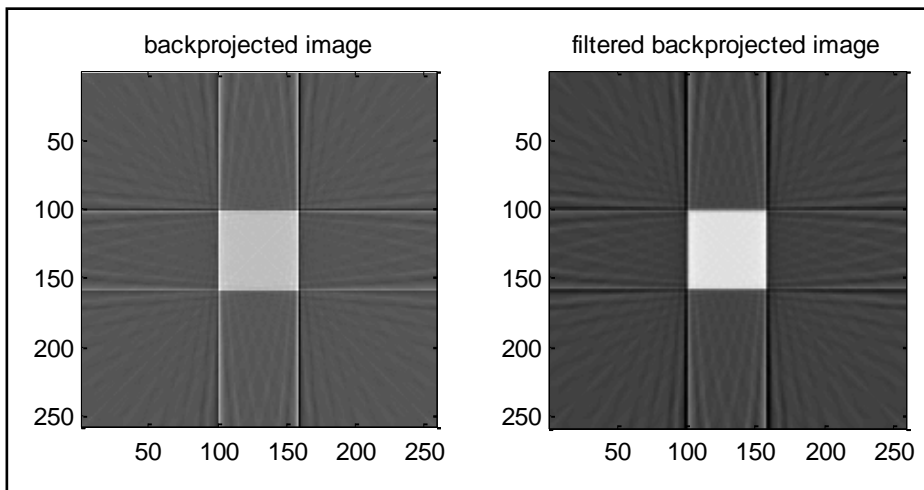
# Example 2: Parallel beam BP

- Run Ex2\_parallel\_back.m
- **Line 32** implements normal BP (no filter)
- **Line 35** implements filtered BP
  - ‘Hamming’ can be replaced with other filter types
    - ‘Ram-Lak’
    - ‘Shepp-Logan’
    - ‘Cosine’
    - ‘Hann’



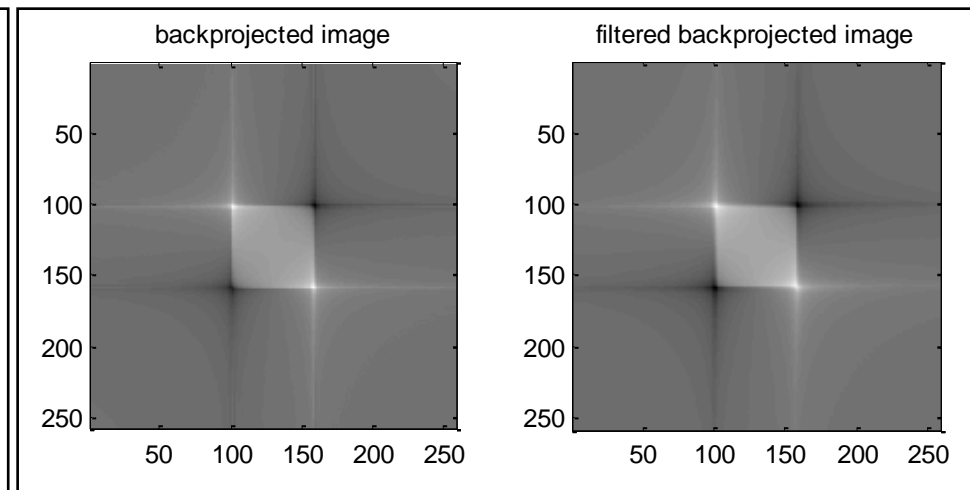
# Example 2: Parallel beam BP

- What happens if you change the maximum  $\theta$  (**Line 23**) or step (**Line 24**)?



max\_theta = 180  
step = 5

**\*The reconstructions  
are less defined  
(discrete steps)\***



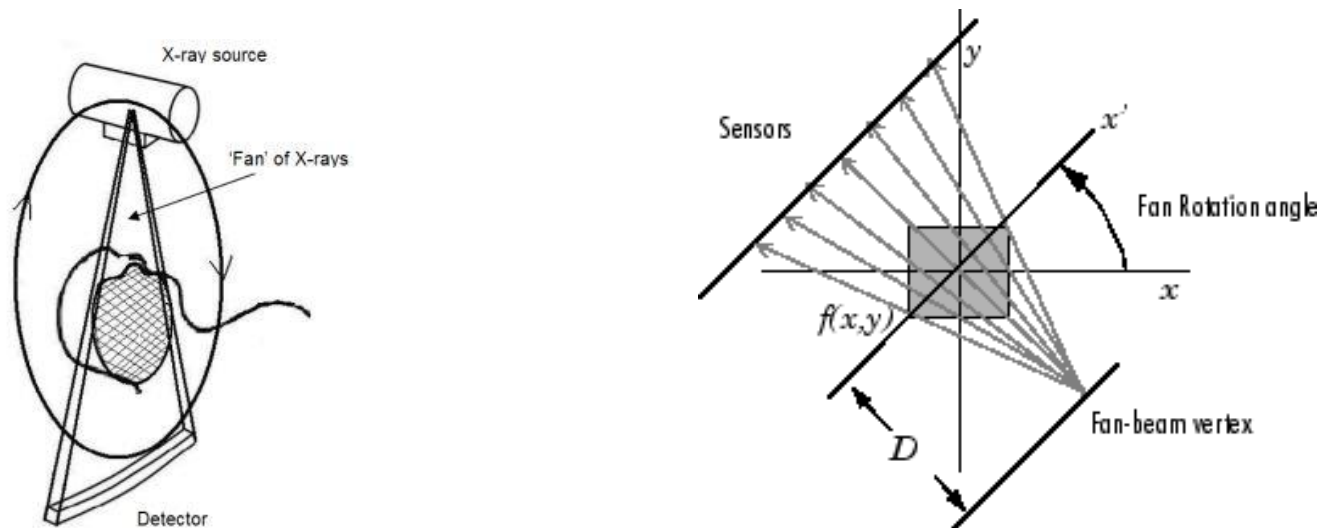
max\_theta = 90  
step = 1

**\*The reconstructions  
have lower resolution  
(less angles)\***

Fan beam CT

# Fan beam: *fanbeam*

- Source beams are projected in a fan shape from a beam vertex
- $D$  is the distance between the fan beam vertex and sample's center of rotation
- Source and detector are rotated at an angle  $\theta$



# Fan beam: *fanbeam*

- Calculates projection data for a specified fan beam geometry
- Rotation angles ( $\theta$ ) fixed at 0 to 360 degrees

```
[F1, sensor_pos1, fan_rot_angles1] = fanbeam(P, D,  
      'FanSensorSpacing', dsensor1);
```

**F1** = output

**sensor\_pos1** = sensor positions

**fan\_rot\_angles1** = fan rotation angles

**P** = input image

**D** = distance to object

***FanSensorSpacing*** = specific property

***dsensor1*** = spacing between sensors

(*FanSensorSpacing*, *dsensor1* are optional)

# Fan beam FBP: *ifanbeam*

- Converts the fan-beam data to parallel beam projections
- Uses the filtered back projection algorithm to perform the inverse Radon transform

```
ifan1 = ifanbeam(F1, D, 'FanSensorSpacing', dsensor1);
```

**ifan1** = output

**F1** = sinogram

**D** = distance to object

***FanSensorSpacing*** = specific property

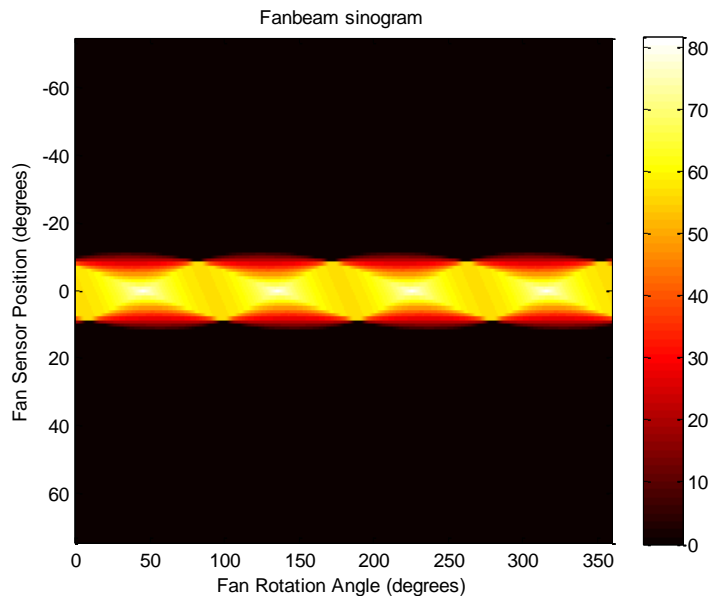
***dsensor1*** = spacing between sensors

(*FanSensorSpacing*, *dsensor1* are optional)

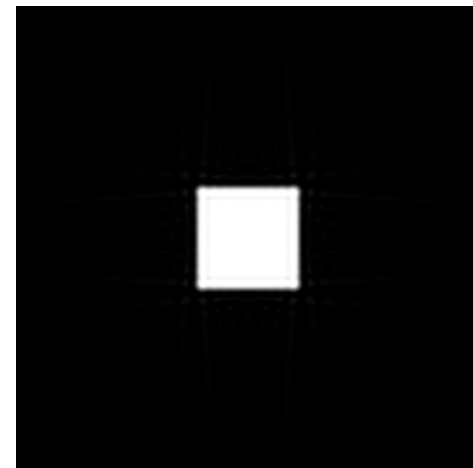
\*Change p\_type (Line 5) to 1, 2, or 3 for different sample types\*

# Example 3: fan beam CT

- Run Ex3\_fanbeam.m
- D (**Line 25**) is slightly larger than half the diagonal distance of image (convention),
- Dsensor = 1 (**Line 26**)



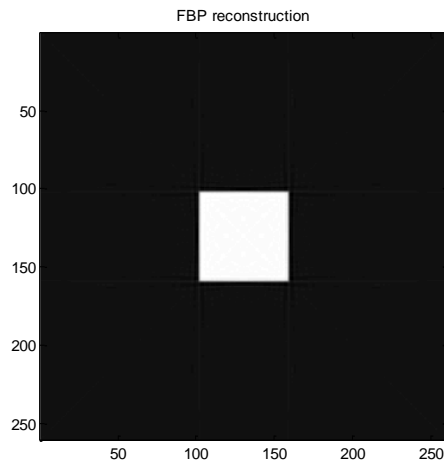
FBP reconstruction



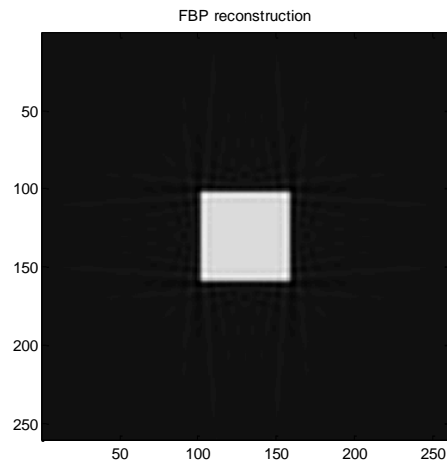


# Example 3: fan beam CT

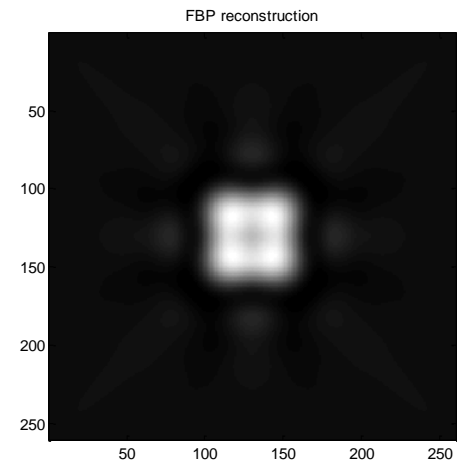
- What happens if you change the sensor spacing (**Line 26**) from 1 to 5? From 1 to 0.5? (D unchanged)



**Dsensor = 0.5**



**Dsensor = 1**



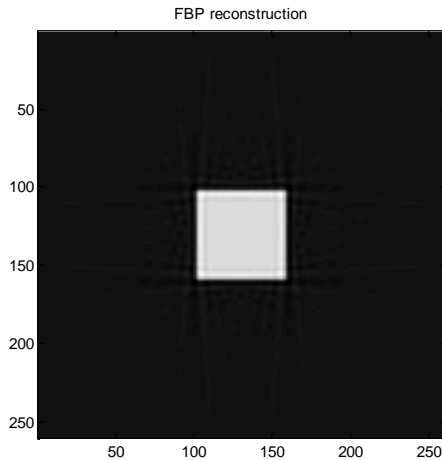
**Dsensor = 5**

**\*The resolution is related to the sensor spacing (high spacing = low resolution)\***

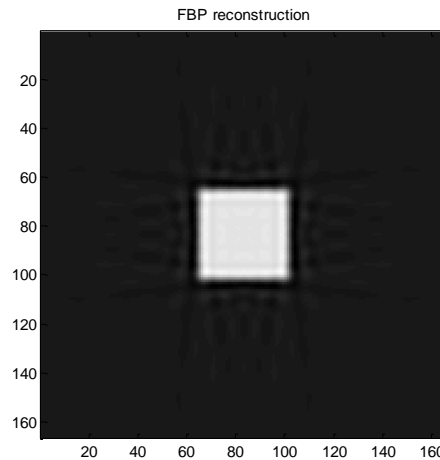
# Example 3: fan beam CT

- What happens if you increase D in **Line 28**?  
(Dsensor = 1)

```
28 - [F, pos, theta] = fanbeam(img, D, 'FanSensorSpacing', Dsensor);
```

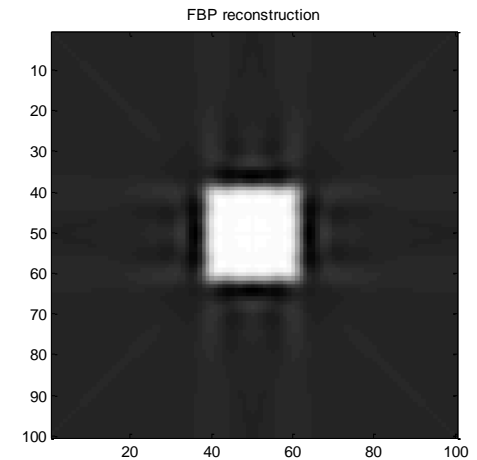


**D = default**  
**(~191)**



**D = 300**

**\*Higher source-object  
distance gives lower  
reconstruction resolution\***



**D = 500**

# BB13 Homework

Read the web page carefully and run the codes on your PC:

<http://www.mathworks.com/help/images/radon-transform.html?refresh=true>

Then write a similar page with an ellipse as your object (instead of the square).

Use the iradon function to reconstruct your ellipse.

**Due Date: Tuesday 3/20 (11:59PM)**

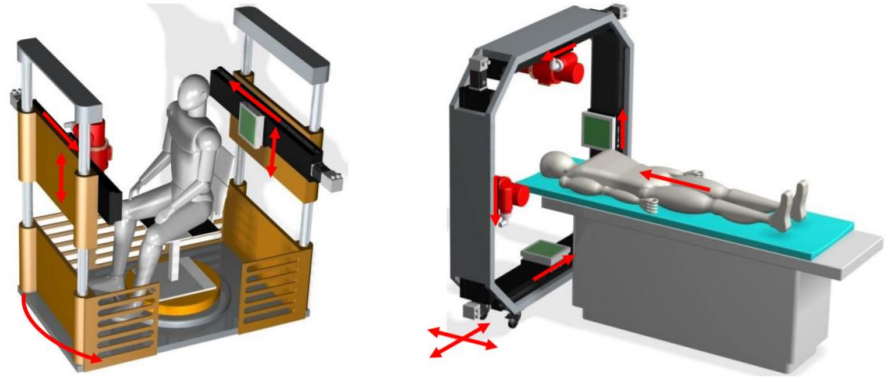
# BB14 Homework

**Green Book**

**1.14, 1.15 and 1.18**

Optional:

As an Art\_X Project, Please Design Portable CT Scanner in an Auto-driving Car, or a CT Scanner for a Novel Application.



**Due Date: Friday 3/23 (11:59PM)**

<https://arxiv.org/ftp/arxiv/papers/1312/1312.6046.pdf>

<http://content.iospress.com/articles/journal-of-x-ray-science-and-technology/xst00453>

Thank you